

Percentages & Work Wrap-Up*

BEGINNERS 01/25/2015

1. The length of a rectangular plot of land was increased by 50% while its width was decreased by 50%. By what percentage did the area change?

2. What is bigger: 26% of 53 or 53% of 26?

*Most problems in this session are taken from "*1001 math problems*" by A. Spivak (in Russian)

3. The length of the first box is 10% more than the length of the second box. The width of the first box is also 10% more than the width of the second box. On the other hand, the height of the first box is 20% *less* than the height of the second box. The volume of the first box is equal to 1000 in³. What is the volume of the second box? Please draw a picture.
4. Abby, Betty and Cindy are running a 100 m race. When Abby was finishing, Betty was 10 m behind her. When Betty was finishing, Cindy was 10 m behind her. How far behind Abby was Cindy when Abby was finishing the race?

5. Alex, Bob and Caden were picking mushrooms. Bob picked 20% more than Alex, and 20% less than Caden. How much more did Caden pick compared to Alex (in percentages)?

6. Jack attends a math circle where over 93% of participants are girls. What is the smallest possible number of students in that circle?

(a) This means that the number of boys is strictly less than ____%.

(b) Finish the problem the problem given that there is at least one boy (Jack) in the math circle.

7. What is the smallest possible number of students in a math circle where the number of girls is more than 40% but less than 50%? We are given that,

$$40\% A < G < 50\% A$$

where A represents all the students and G represents the girls.

- (a) Rewrite the above inequality using fractions instead of percentages.
- (b) Can it work when $G = 1$? Explain.
- (c) Find the smallest possible number of girls.
- (d) Finish the problem and find the smallest possible number of students in the math circle.

8. The first mill makes 60 bags of flour in 24 hrs. The second one makes 54 bags of flour in the same period. The third makes 48 bags of flour in a day. How long would it take for all three mills working together to make 81 bag of flour?

9. The first snowplow machine can clean a street in 1 hr. The second one can do this in 45 minutes. The two machines worked simultaneously for 20 minutes. After this, the first machine broke down. How long will it take for the second machine to finish cleaning the street?

10. Make up your own interesting and challenging problem about percentages. Write down the problem in complete sentences and provide a solution.

11. Make up your own interesting and challenging problem about combined work (people performing a task (e.g., working, eating something, etc.) together, or pipes filling a pool, etc.). Write down the problem in complete sentences and provide a solution.

12. Find the number x such that

$$x \cdot x = 12345678987654321.$$

(*Hint:* Use vertical multiplication to explain your answer).

13. Find two consecutive number integers x and $x + 1$ such that if you multiply one by the other you get $\underbrace{11\dots11}_{100} \underbrace{22\dots22}_{100}$. That is,

$$x \cdot (x + 1) = \underbrace{11\dots11}_{100} \underbrace{22\dots22}_{100},$$

where the number on the right is written with 100 digits 1 and 100 digits 2. Justify your answer.

14. (Math Kangaroo) Four teams took part in a soccer tournament. The rules were: (a) each team plays against each of the other teams exactly once, and (b) a team gets 3 points for winning, 0 points for losing and 1 point if there is a tie. At the end of the tournament, the teams had respectively 5 points, 3 points, 3 points and 2 points. How many of the games ended in a tie?
15. (Math Kangaroo) Snow White lined up the Seven Dwarfs from shortest to tallest. She divided between them 77 berries which they had picked in the forest. The shortest dwarf got a certain number of berries, the next one got one berry more, and so on. How many berries did the tallest dwarf get?