If you move counterclockwise by angle $\theta$ around the unit circle in the plane, the $x$ coordinate of where you end up is $\cos \theta$, and the $y$ coordinate of where you end up is $\sin \theta$.

**Problem 1**

What are $\sin 0^\circ$, $\sin 270^\circ$, $\sin 360^\circ$, $\cos 90^\circ$, $\cos 180^\circ$, and $\cos 360^\circ$?
When doing trigonometry, we usually use a different unit for angles; instead of degrees, where a full circle is $360^\circ$, we use radians, where a full circle is $2\pi$ radians. This system has the advantage that, when going $\theta$ radians around the unit-radius circle, the length of the arc covered is also $\theta$.

**Problem 2** Fill in these boxes with the angle in radians, and label the endpoints with $(\cos \theta, \sin \theta)$
Problem 3

Consider any right triangle, with angles $\theta, \pi/2, \text{and } \pi/2 - \theta$ (remember, we’re working in radians, and $\pi/2$ radians is $90^\circ$). If the leg opposite $\theta$ has length $A$, the leg adjacent to $\theta$ has length $B$, and the hypotenuse has length $H$, describe $\sin \theta$ and $\cos \theta$ in terms of $A, B,$ and $H$.

Problem 4  Show that the following formulas hold

(Hint: Use the unit circle)

$\sin^2 \theta + \cos^2 \theta = 1$

$\sin(\theta + \pi/2) = \cos \theta$

$\cos(-\theta) = \cos \theta$

$\sin(-\theta) = -\sin \theta$
We define the tangent function, \( \tan \theta \), as \( \tan \theta = \frac{\sin \theta}{\cos \theta} \)

**Problem 5**

If \( \sin \theta = \frac{3}{5} \), what are \( \cos \theta \) and \( \tan \theta \)?

If \( \tan \theta = 7 \), what are \( \cos \theta \) and \( \sin \theta \)?

If \( \cos \theta = \frac{5}{13} \), what are \( \sin \theta \) and \( \tan \theta \)?

**Problem 6**

Use a geometric argument to show that \( \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha \)
Problem 7

Use the equations you’ve already shown to prove the following:
\[
\sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha
\]

\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
\]

\[
\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta
\]

Problem 8

Show that:
\[
\sin 75^\circ = \sin \frac{5\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4} \quad \text{and}
\]
\[
\cos 75^\circ = \cos \frac{5\pi}{12} = \frac{\sqrt{6} - \sqrt{2}}{4}
\]
Problem 9  Uniformization

Show that:

\[ \sin \theta = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \quad \cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \quad \tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} \]

Problem 10  Pythagorean Triples via Uniformization

If \((a, b, c)\) is a Pythagorean Triple, i.e. a set of positive integers such that \(a^2 + b^2 = c^2\), then there is a right triangle with legs \(a/c\) and \(b/c\), and hypotenuse 1. Thus the sine, cosine, and tangent of each acute angle of this right triangle are rational numbers. So, set \(\tan \frac{\theta}{2} = \frac{p}{q}\) for positive integers \(p, q\). Then, find the corresponding pythagorean triple.