Number Theory (Part 3)

#1 Take the number $100! = 1 \times 2 \times 3 \times \ldots \times 100$. Add up all the digits of this number. For the number you get, do the same thing (add up all the digits). Continue until you get a one digit number. What number will you get?

#2 Find the smallest possible number written with only digits 1 and 0 and such that it is divisible by 225.

#3 Show that the number $\frac{n^3 - n}{6}$ is an integer for any integer $n$. 
#4 Find \( x \) and \( y \) given that \( \gcd(x, y) = 5 \) and \( \text{lcm}(x, y) = 30 \).

#5 Show that a number written with digits \( xyzw \) (in this order) is divisible by 99 if and only if the sum of the number written as \( xy \) and \( zw \) is divisible by 99.