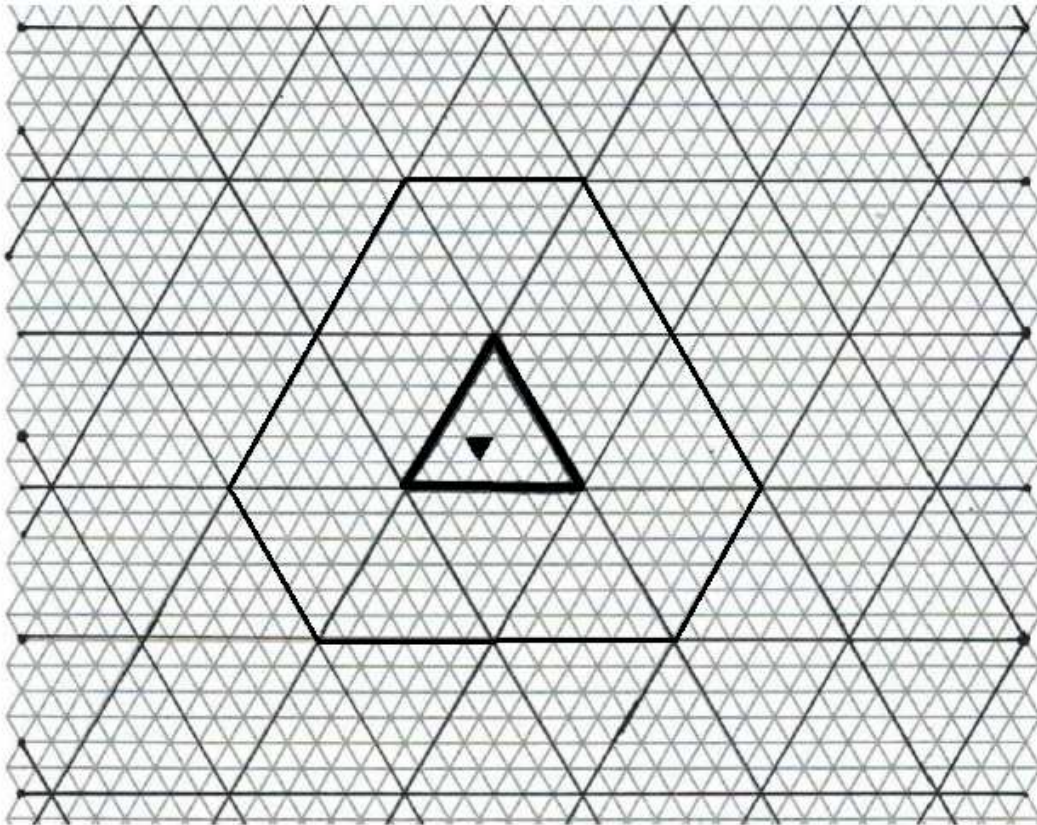


SYMMETRY, MIRRORS & MISCELLANEOUS PROBLEMS

BEGINNERS 12/7/2014

Warm Up

The large, boldfaced equilateral triangle represents the 3-mirror system of a kaleidoscope. The shaded triangle represents the object inside your kaleidoscope. Reflect the shaded triangle in all the mirrors, then reflect all the reflections to generate an overall pattern.



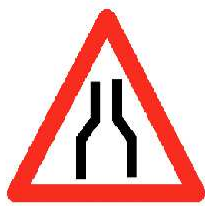
Roadside Symmetry.

1. Road signs gives us different kind of information. Many road signs exemplify some symmetry.

(a) Determine which signs below have symmetry. For the signs with symmetry, draw their axis or axes of symmetry. For the signs without symmetry, circle the parts of the sign that are not symmetrical. You can use your Reflect-It mirror to help you find the axes of symmetry.



SIDE ROAD



ROAD NARROWS



CAMPGROUND



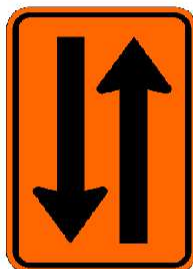
STOP



TRAFFIC CIRCLE



ROAD HAZARD



TWO-WAY TRAFFIC



TRAFFIC LIGHT AHEAD



STOP AHEAD

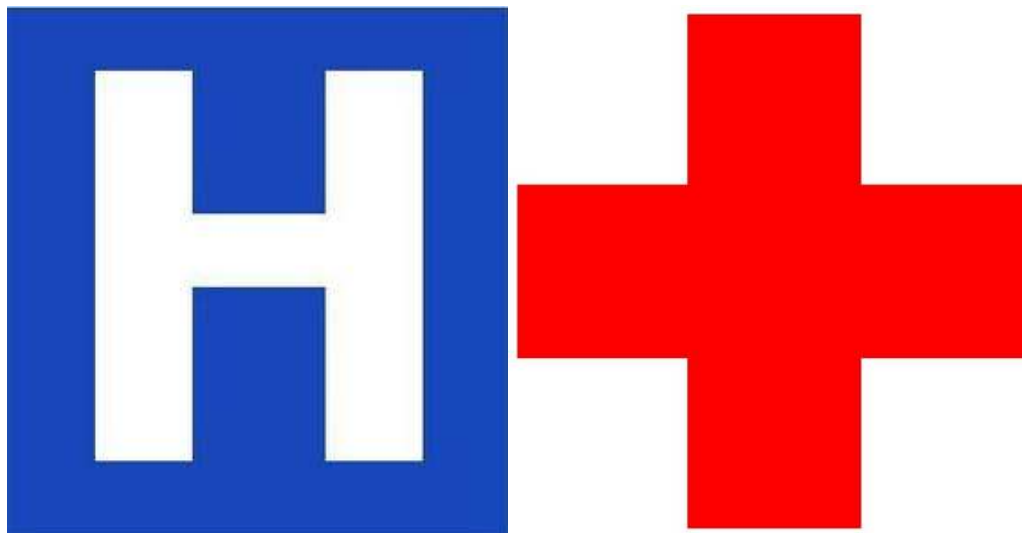


CROSS ROAD

(b) The sign below indicates the way to a hospital. You can use your Reflect-It mirror to generate the whole picture using only part of the image because of symmetry. Use the Reflect-It mirror to find different ways to generate the whole hospital sign. Can multiple angles generate the complete sign? Find the smallest angle that generates the complete sign? Draw onto your hospital sign where you placed your Reflect-It mirror.



(c) It is possible to use parts of one image to create another using your Reflect-It mirror. Using your Reflect-It mirror, can you use the image of the hospital sign to create the red-cross sign below? If so, draw onto your hospital sign where you placed your Reflect-It mirror to do so.



Toothpick Polygons.

2. All the angles of regular polygons are equal in measure and all the sides are equal in length.

(a) Position your Reflect-It mirror to form a 60° angle. Place the toothpick between the faces of the mirror until you generate an equilateral triangle. Describe your results and draw a sketch of how you placed the toothpick.

(b) Reposition your Reflect-It mirror to form a 45° angle. Place the toothpick between the faces of the mirror until you generate a square. Describe your results and draw a sketch of how you placed the toothpick.

(c) What mirror setting should you use to form a hexagon? Describe your results and draw a sketch of how you placed the toothpick?

Miscellaneous Problems.

The following problems come from *Invitation to a Mathematical Festival* by Ivan Yashchenko.

1. In one country's Parliament, there are two houses with an equal number of members each. All members took part in a vote on an important issue and none abstained. When the chairman announced that the decision was made by margin of 23 votes the leader of the opposition declared that the results of the vote were forged. How did he know?

2. Forty-eight blacksmiths are supposed to shoe 60 horses. What is the shortest amount of time this will take if each blacksmith spends five minutes on each horseshoe? A horse cannot stand on just two legs, so at most one blacksmith can work on a horse at any time.

3. (a) Cover the plane fully with congruent pentagons. The polygons do not have to be regular.

(b) Now, cover the plane fully with congruent heptagons (7-sided polygons). The polygons do not have to be regular.

4. Mark 6 points on the plane so that for every point there are exactly three other points at a distance of 1 from it.

5. Color a plane in three colors so that there are points of no more than two colors on each straight line and all three colors are used.

6. The sieve of Erasthóthenes is a Greek algorithm used to find all the prime numbers. Using the table below, circle the smallest prime number 2. Then cross out all multiples of 2. The closest number to the right of 2 that has not been crossed out should be next smallest prime number. Circle it, then cross out all its multiples. Repeat this process.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	30
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

(a) Put 3 prime numbers in a row so that the differences between adjacent numbers in each row are equal. The prime numbers do not have to be consecutive prime numbers.

(b) Put 4 prime numbers in a row so that the differences between adjacent numbers in each row are equal.

(c) Put 5 prime numbers in a row so that the differences between adjacent numbers in each row are equal.

7. Every seventh mathematician is also a philosopher, and every ninth philosopher is also a mathematician. Are there more philosophers or more mathematicians? (Hint: Use a Venn diagrams.)

8. Solve the following cryptarithms.

(a)

$$\begin{array}{r} \text{NUMBER} \\ + \text{NUMBER} \\ \hline \text{PUZZLE} \end{array}$$

b)

$$\begin{array}{r} \text{CLOCK} \\ + \text{TICK} \\ + \text{TOCK} \\ \hline \text{PLANET} \end{array}$$