MIRRORS AND KALEIDOSCOPES

BEGINNERS 11/23/2014

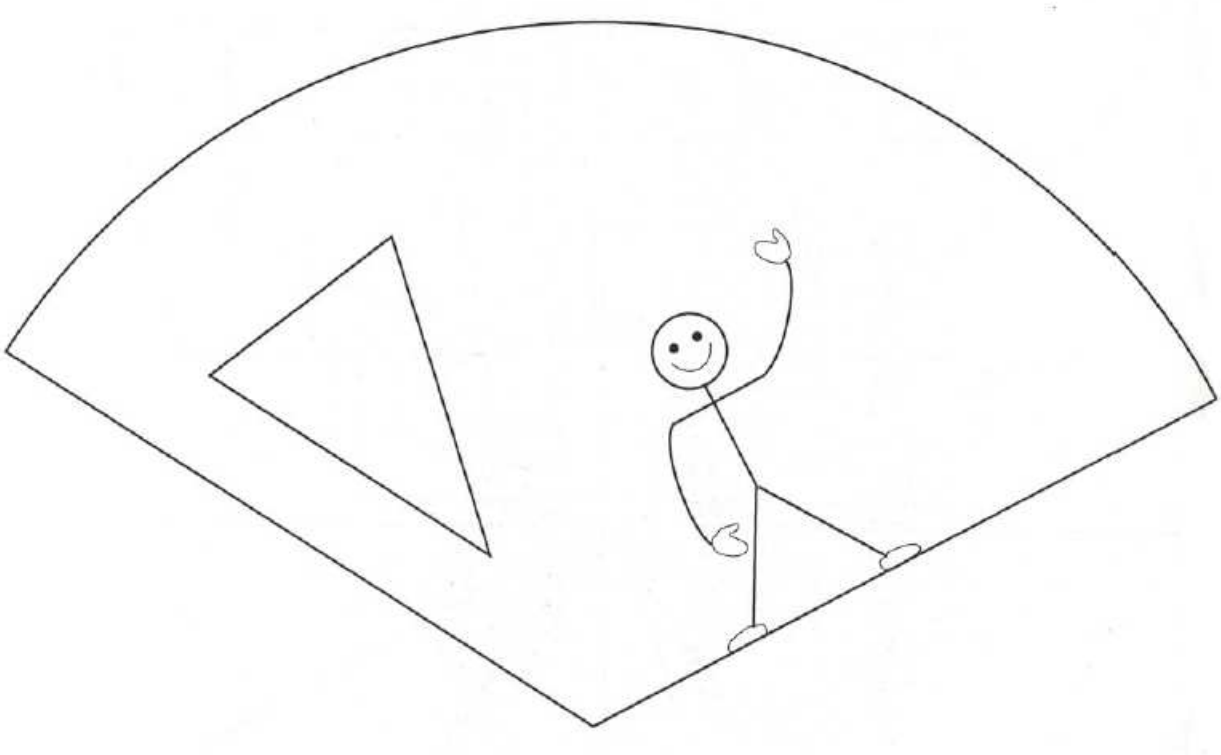
Explore Reflect-It Mirror.

1. Can you make the following angles using your Reflect-It Mirror?

- 180°
- 90°
- 60°
- 45°
- 30°

2. What is the degree measure between two "thin" marks on the mirror stand?

3. A mirror does not *reproduce*, or copy, the original object or design. A mirror *reflects* the original object or design. To understand what it means, let's look at the Stick Figure Wedge.



(a) Examine the figure on the wedge. From the perspective of the stick figure, which arm is raised?

(b) Position the Reflect-It mirror, forming a 120° angle around the wedge. Examine the reflection of the stick figure in the left mirror face. From the perspective of the reflection of the stick figure, which arm appears to be raised now?

(c) In your own words, explain the difference between the words $reflect$ and $reproduce$. Give an example of each.

(d) Measure the distance between the drawing of the triangle and the edge of the wedge of paper.

(e) Estimate the distance betwen the original triangles and its image in the right mirror face. Why does the distance appear to be twice the original measurement?

2-Mirror System

The Reflect-It Mirror is a 2-mirror system. It produces a central image in the middle of the viewing field. The image is circular and is a reflection of the object viewed. The angle of the Reflect-It Murror determines the number of reflections.

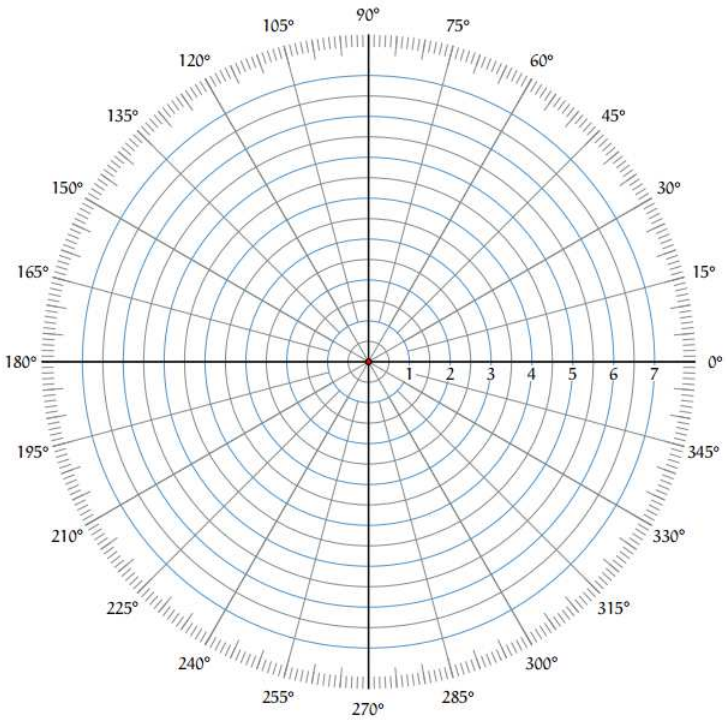
1. Place an object (e.g. eraser) in the center of the viewing field of your Reflect-It Mirror.

(a) How many reflected images do you get depending on the angle? Count the orginal object as one of the images.

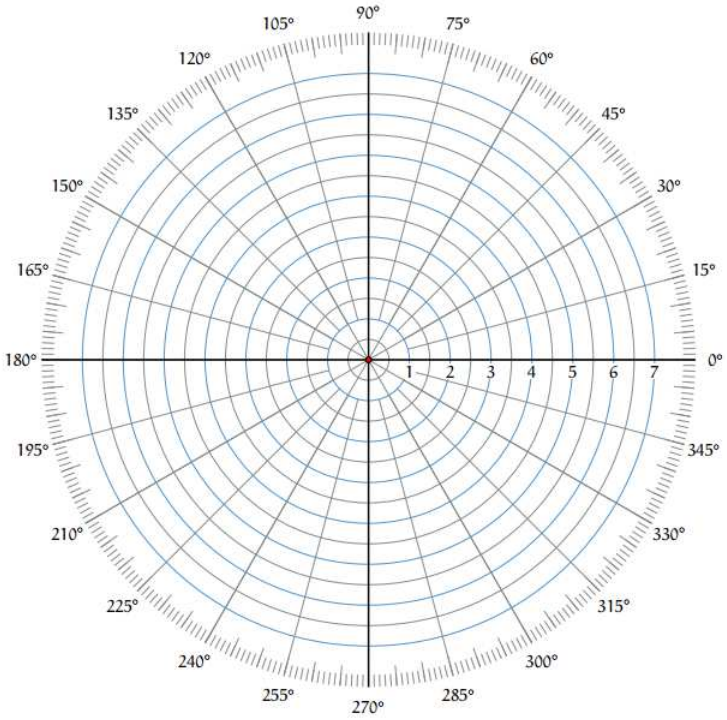
Angle	Total Number of Images
$180^\circ = \frac{360^\circ}{}$	2
$= \frac{360^\circ}{}$	3
$= \frac{360^\circ}{}$	4
$= \frac{360^\circ}{}$	5
$= \frac{360^\circ}{}$	6
$= \frac{360^\circ}{}$	7
$= \frac{360^\circ}{}$	8
$= \frac{360^\circ}{}$	10
$= \frac{360^\circ}{}$	12

(b) Using the polar coordinates below, draw your mirror angle, your object, and its reflections. Place your object in the middle of your angle. When drawing, you can simplify your object as star. (Hints: To draw your angle, use the ray connecting the center with the 0° as one of the sides.)

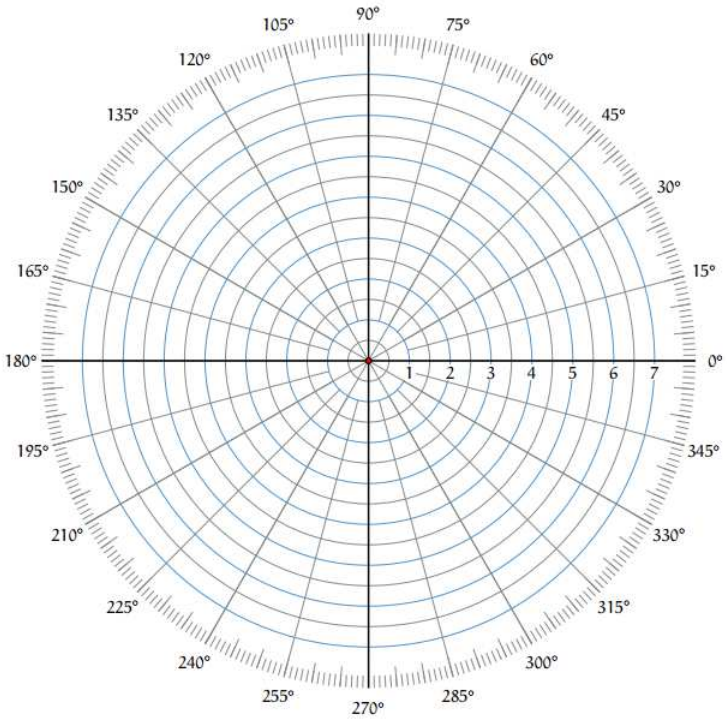
- Mirror angle: 180°, Total number of images:



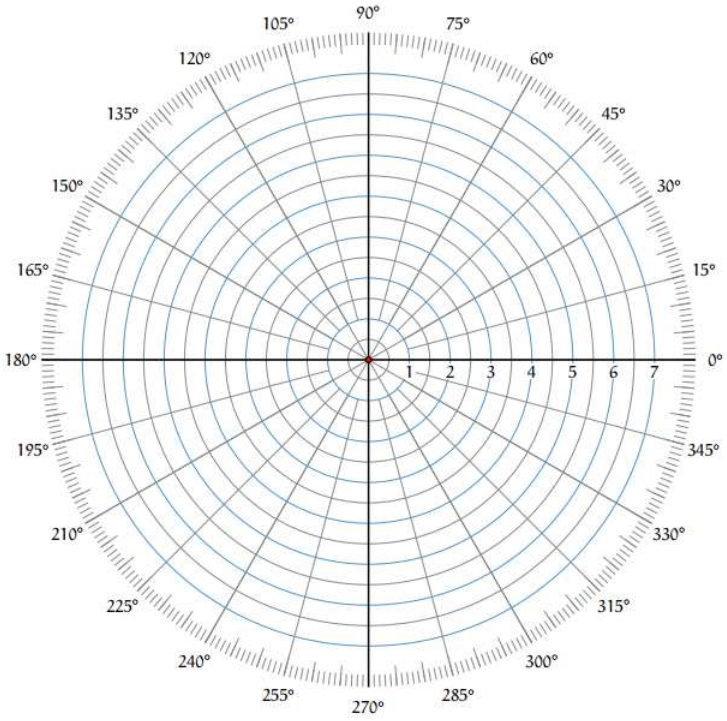
- Mirror angle: 120°, Total number of images:



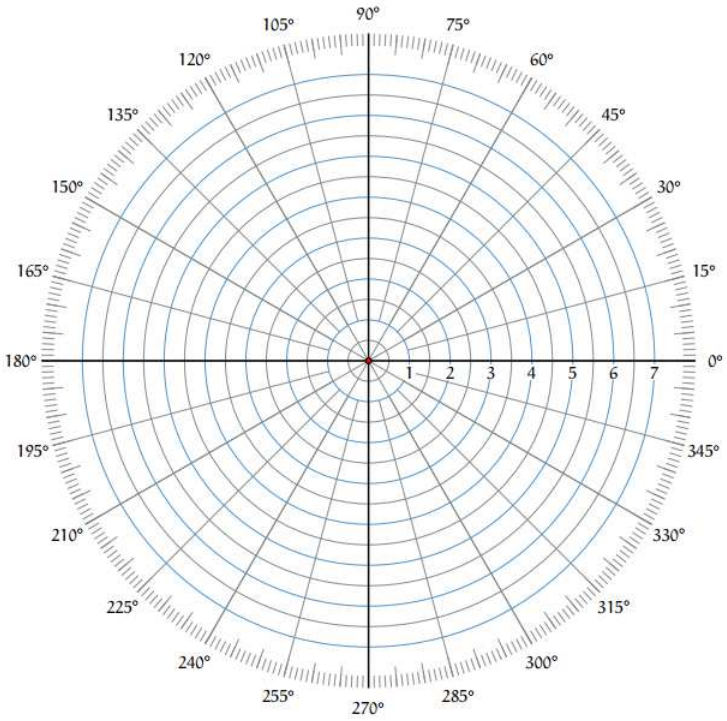
- Mirror angle: 90°, Total number of images:



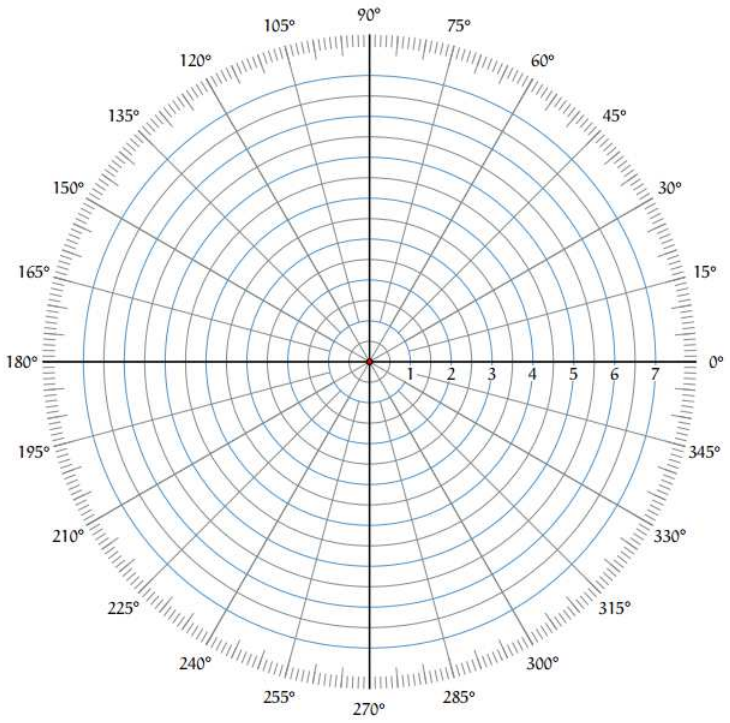
Mirror angle: 60°, Total number of images:



Mirror angle: 45°, Total number of images:



Mirror angle: 30°, Total number of images:

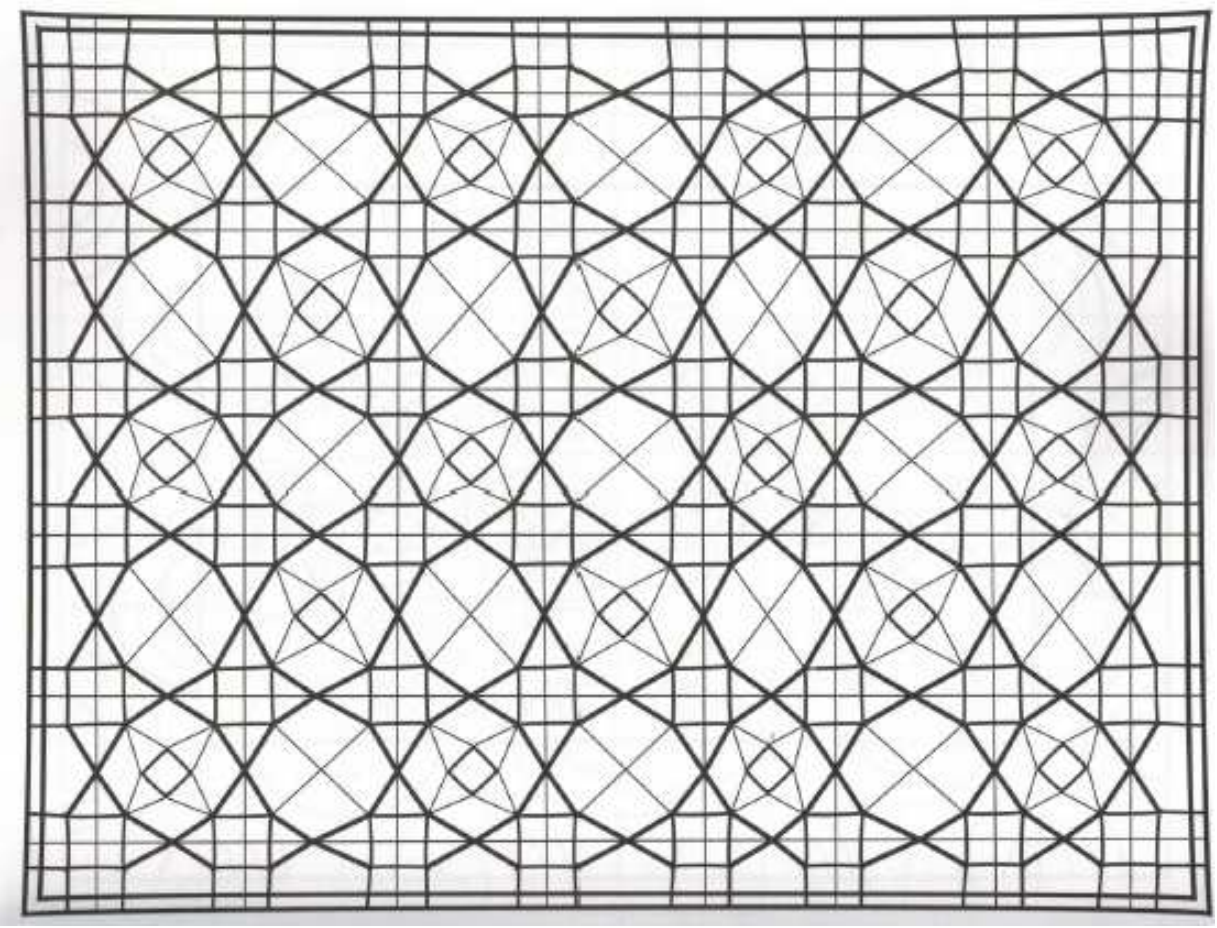


2. (a) What happens to your image when you open your mirror to 135°? Explain your observation.

(b) What is the rule to determine which angles produce a closed design around the vertex?

(c) Could you reflect your object infinitely using the Reflect-it Mirror? If not, what could you do to reflect your object infinitely?

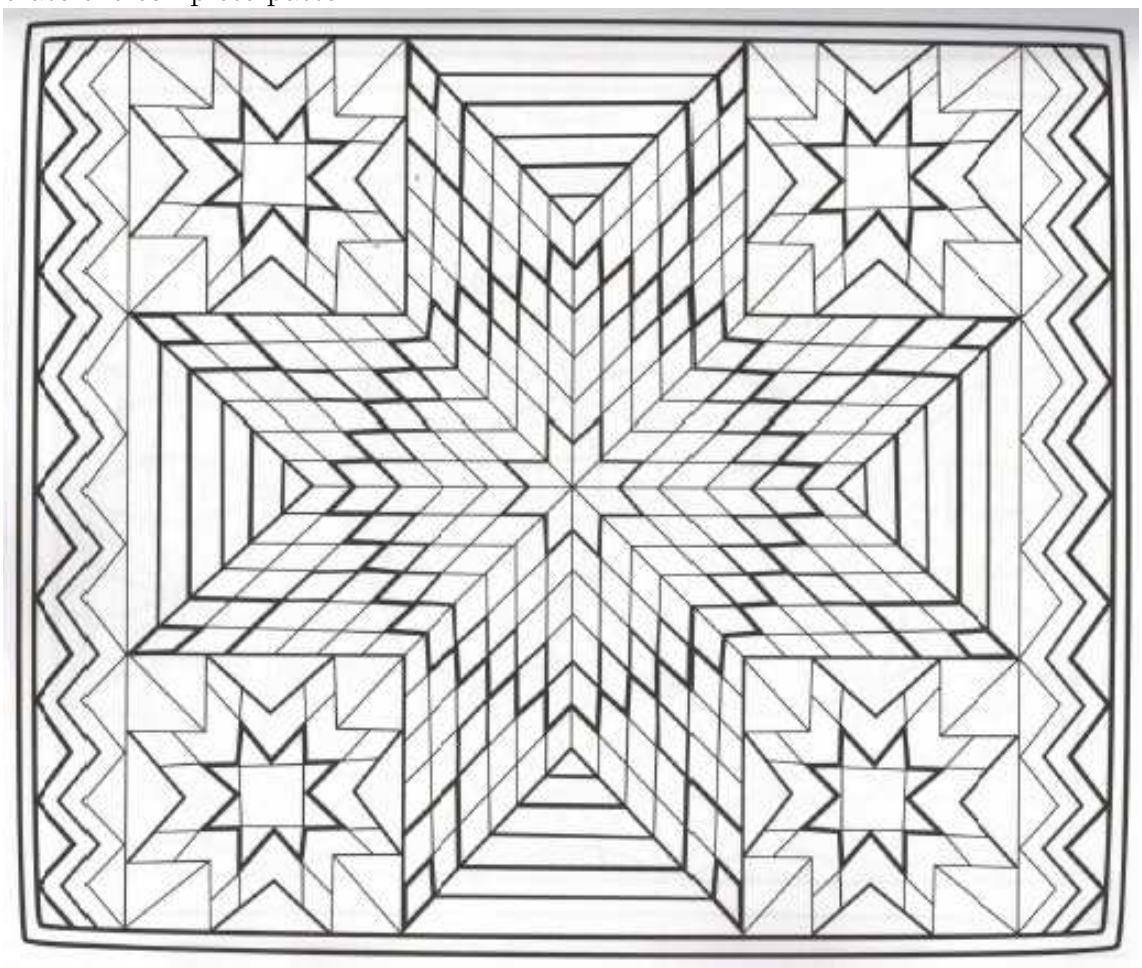
3. For the complex pattern below, determine where to place your Reflect-It mirror to generate the complete pattern.



(a) Is it possible? Explain.

(b) If it is possible, can multiple angles generate the complete image? Find the the smallest angle opening that generates the complete image?

4. For the complex pattern below, determine where to place your Reflect-It mirror to generate the complete pattern.



(a) Is it possible? Explain.

(b) If it is possible, can multiple angles generate the complete image? Find the the smallest angle opening that generates the complete image?

5. Now try it with your colored rosette (last week's challenge problem).

Kaleidoscopes.

The word "kaleidoscope" comes from Ancient Greek and means "observation of beautiful forms." Kaleidoscopes are devices that produce a beautiful, overall pattern from the original design and the reflected images of it in the mirrors. The crucial element of a kaleidoscope is the mirror system. Different types of kaleidoscopes depend on the variation of three factors: mirror shapes, number of mirrors used, and the angle at which the mirrors are joined.

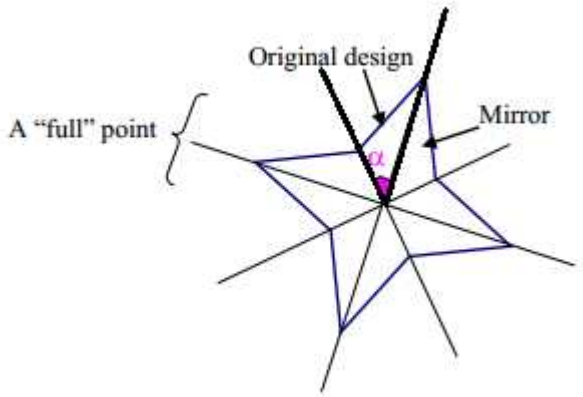
There are four basic componenents in all kaleidoscope designs:

- Eye piece: with variation of the size and shape of the lens
- Body: the external housing (with optional stands, rotating gears, etc.)
- Mirror system: 2- or 3- mirror systems configurations are the most popular
- Object chamber: different materials can be used for viewing (beads, pebbles, etc.)

Good Kaleidoscope Angle.

A good kaleidoscope angle is one that produces a "star" as an overall pattern. The star has n *full points*, each consisting of a copy of the original design plus its mirror image. The n points fit together at the point where two mirrors meet, without overlap, to form the overall pattern.

If the star has n points, and the points fit around the vertex without overlapping, then $360^\circ = n \cdot 2\alpha$, $\alpha = \frac{180^\circ}{n}$. α is the angle of the orginal design. A full point is of angle 2α.



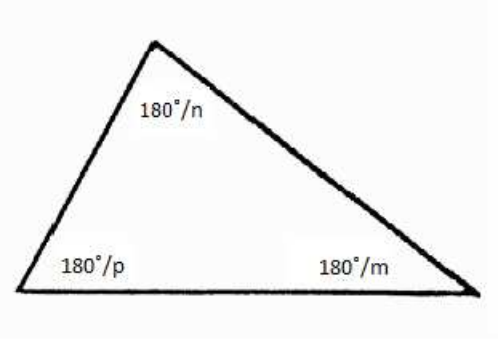
1. (a) Draw a full point.

(b) Draw not a full point.

2. A good triangular kaleidoscope must have three good kaleidoscope angles, $\frac{180^\circ}{n}$, $\frac{180^\circ}{m}$ and $\frac{180^\circ}{p}$, where n, m and p are whole numbers.

In order to form a triangle, the sum of these angles must add up to 180°:

$\frac{180^\circ}{n} + \frac{180^\circ}{m} + \frac{180^\circ}{p} = 180^\circ$ simplified as $\frac{1}{n} + \frac{1}{m} + \frac{1}{p} = 1$



Let us determine all the good triangle kaleidoscope angles.

(a) Assume $n \leq m \leq p$. Can $n = 1$? Explain.

$n \geq$

(b) If $n = 2$, what is $\frac{1}{m} + \frac{1}{p}$?

(c) When $n = 2$, what are all the possible values for m and p, and their corresponding angles? Sketch the corresponding triangles for each of your solutions, label the angles.

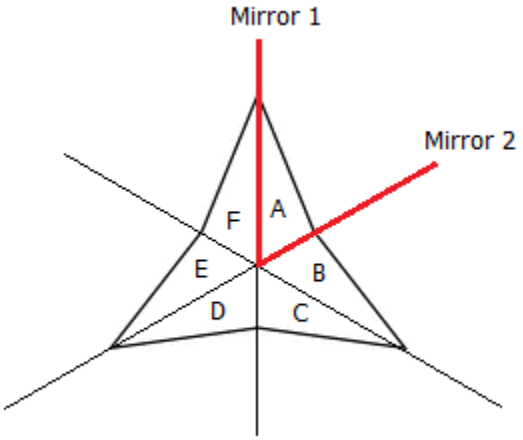
(d) If $n = 3$, what is $\frac{1}{m} + \frac{1}{p}$?

(e) When $n = 3$, what are the only possible values for m and p, and their corresponding angles? Sketch the corresponding triangles for your solution, label the angles.

In conclusion, there are only _____ possible good triangle kaleidoscope configurations.

Kaleidoscope Reflections.

1. The following diagram explains how a 2-mirror kaleidoscope with 60° between the two mirrors, producing 6 images with an overall 3-point star pattern.



(a) A reflects onto ___ in mirror 1.

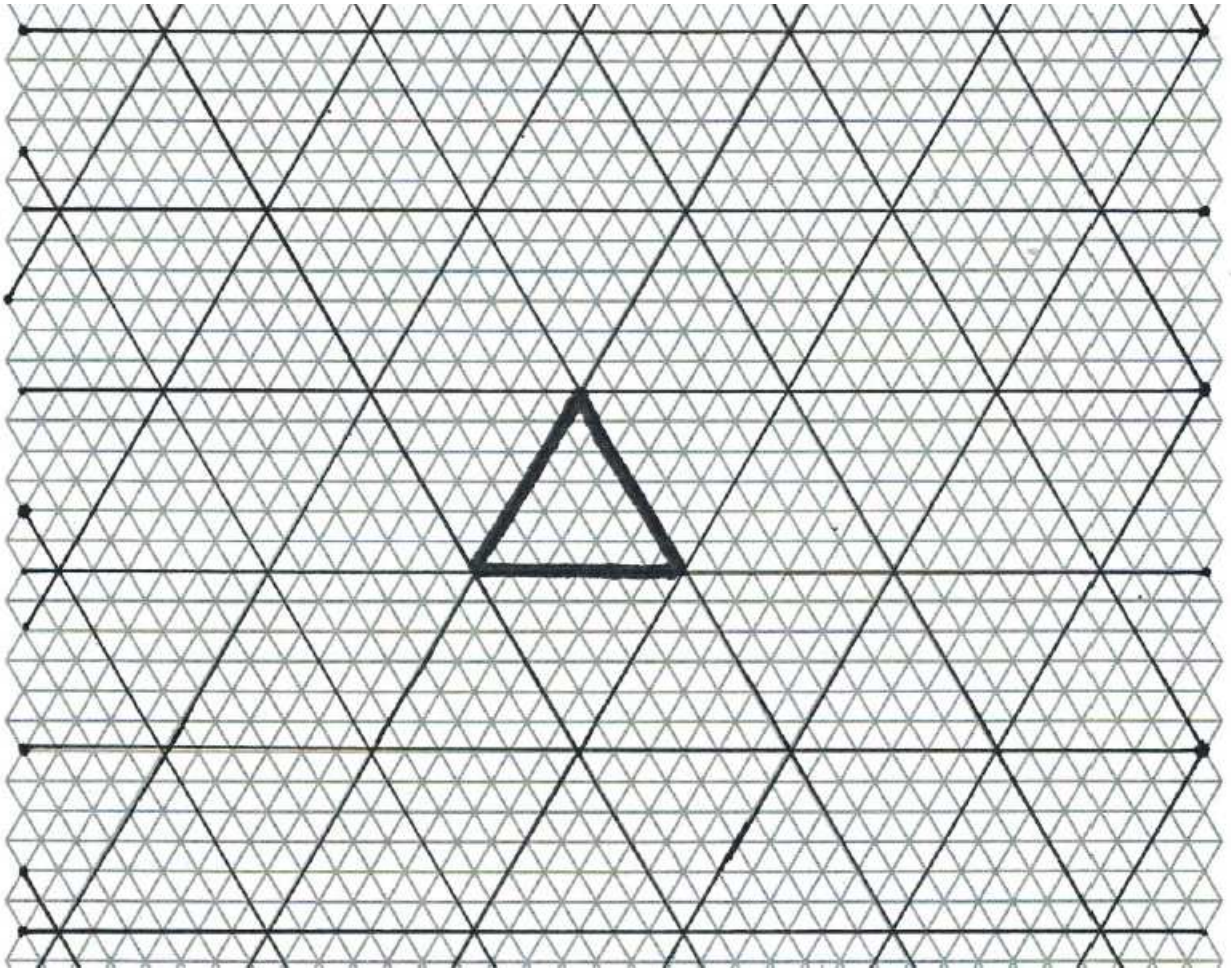
(b) A reflects onto B in mirror ___.

(c) C reflects onto F in mirror ____.

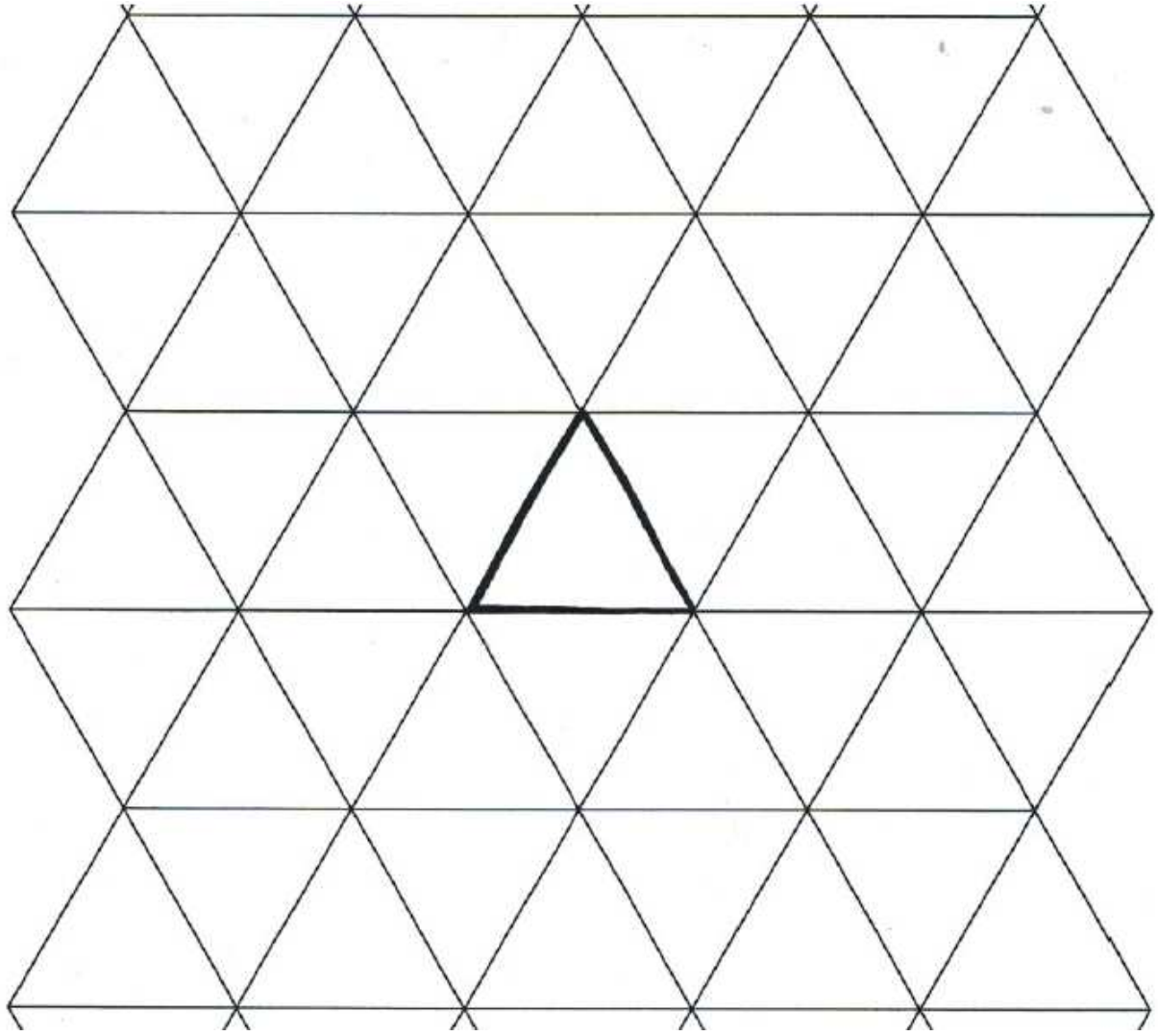
(d) B reflects onto ___ in mirror 1.

A 2-mirror image known as a *mandala* can be extended into a 3-mirror image by adding a third piece of mirror. The image is repeatedly reflected, such that a faceted pattern is formed. Let's practice generating patterns using all three possible 3-mirror (triangle) kaleidoscope systems.

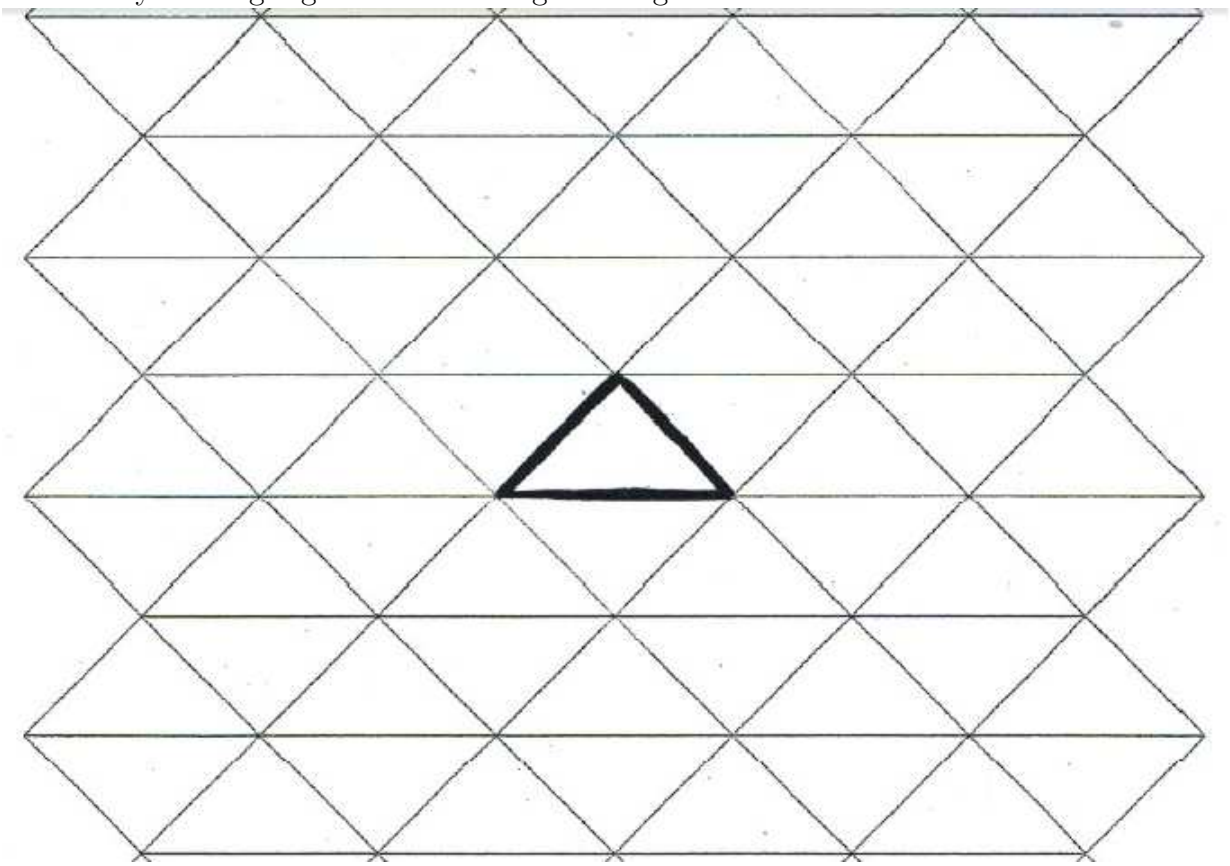
3. Color the small triangle of your choice in the bolded triangle. It represents the object inside your 3-mirror kaleidoscope. Now, reflect it accordingly to generate an overall pattern.



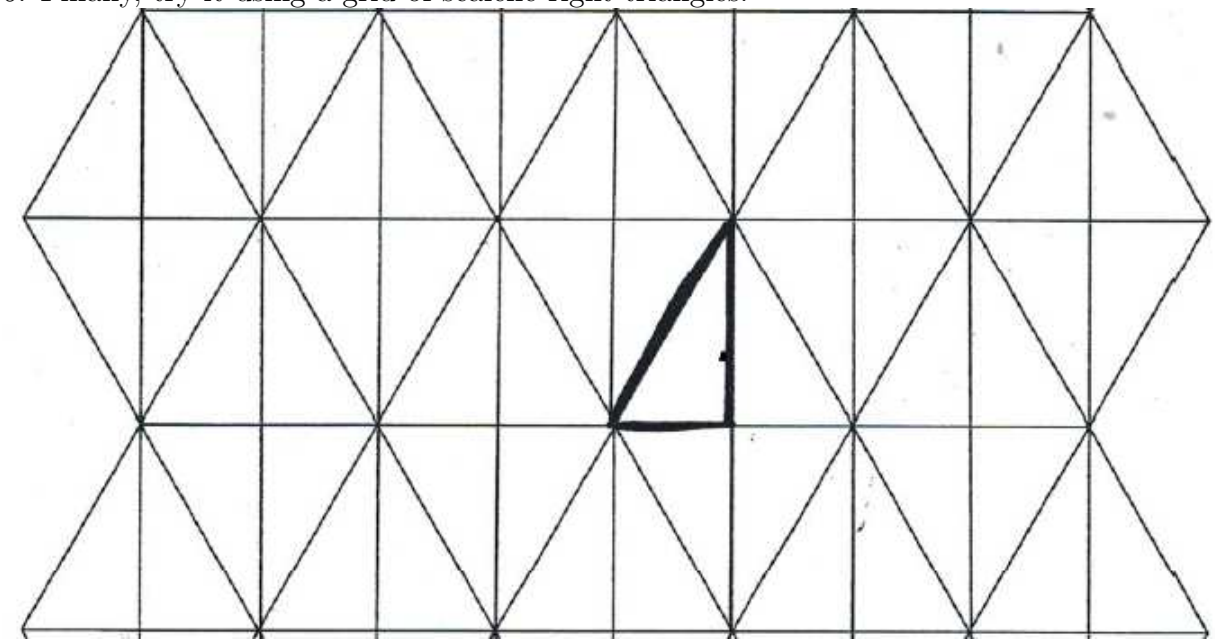
4. Draw the letter of your choice in the bold triangle. If the bold edges are your mirrors, reflect your letter accordingly to generate a 3-mirror kaleidoscope pattern with an equilateral triangle configuration.



5. Now try it using a grid of isoceles right triangles.



6. Finally, try it using a grid of scalene right triangles.



7. How do the patterns in the three differents grids (4-7) differ?