Coins and Chessboards

June 1, 2012

1. Return of Negations!

   Remember that last time we learned how to create an opposite statement of a given statement. Fill in the opposite statements in the chart below:

Statement	Opposite Statement
All people are great writers.	
Some books do not have pictures.	
All people don't like dogs.	
Some cats do not have fur.	

2. Suppose that a statement is false. One way to show that a statement is false is to provide a *counterexample.* Write down the opposite statement and provide a counterexample to the following false statements:

   (a) Statement: All people have straight hair.

   Counterexample:

   Opposite statement:

   (b) Statement: All shoes have laces.

   Counterexample:

   Opposite statement:

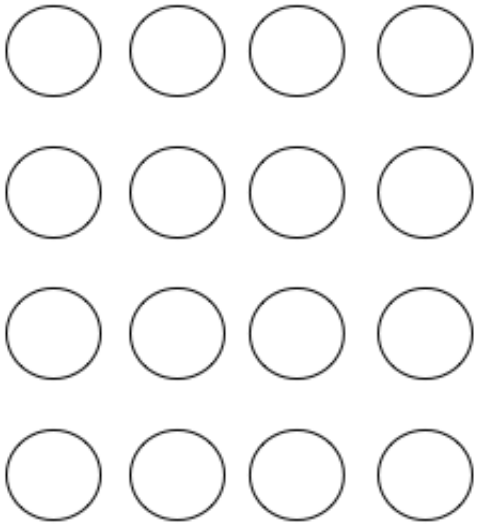
(c) STATEMENT: There are no seven year old students in the math circle.

COUNTEREXAMPLE:

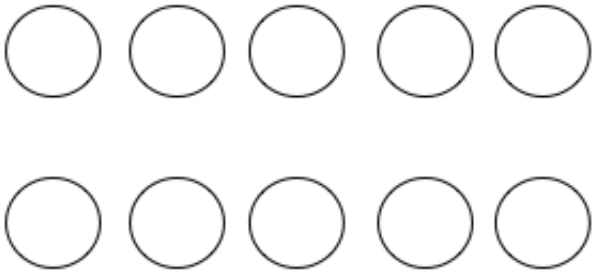
OPPOSITE STATEMENT:

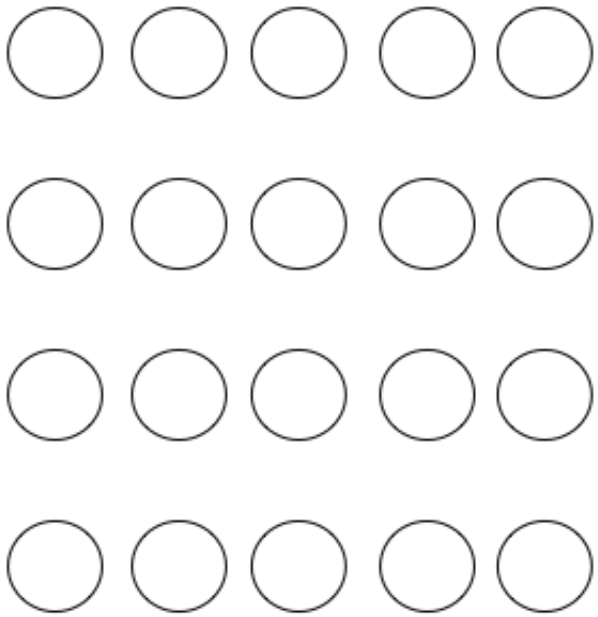
3. There are 4 coins in a row. Let's number them 1 through 4. You are allowed to switch any two coins that have exactly one coin between them. (For example, you can switch the first and the third coin).

(a) Can you reverse the order of the coins by repeating this operation many times? If yes, show how. If not, explain why not.

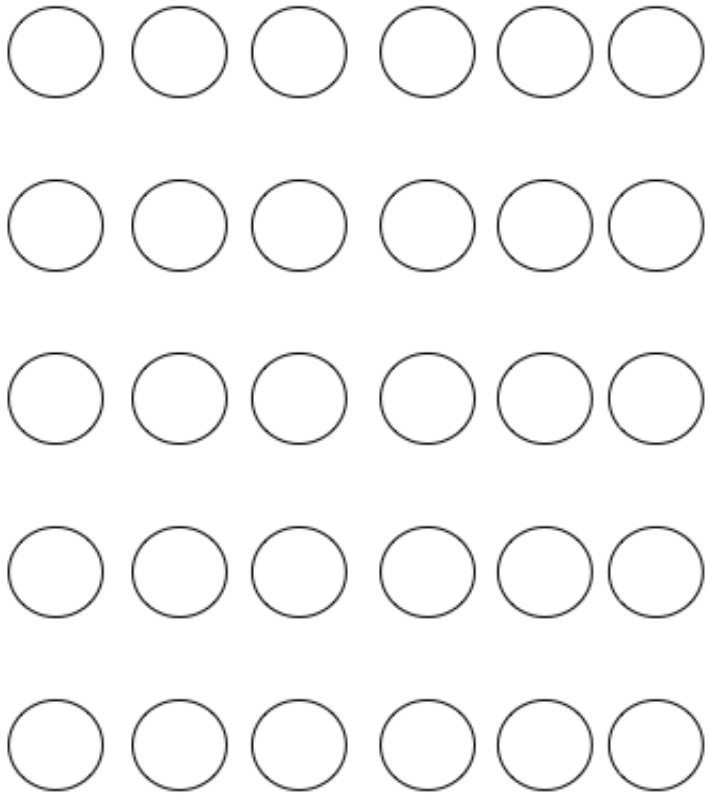


(b) Suppose there are 5 coins in a row numbered 1 through 5. By repeating the same operation (switching two coins which have exactly one coin between them), can you reverse the order of the coins?





(c) Suppose there are 6 coins in a row numbered 1 through 6. Can you reverse the order of the coins now?



(d) Can you reverse the order of the coins if there are 1001 coins?

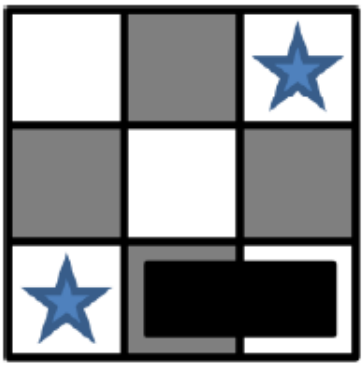
What if there are 2012 coins?

(e) For what numbers of coins can the order be reversed? Explain.

4. Nina has a bag of dominos that she wants to put on chess boards of various sizes. When a domino is placed on the chessboard, it covers two squares that touch each other along a side. Nina wants to place all the dominos on the chessboards so that the lower left and upper right corners are left empty and all other squares are covered by the dominos. The two corners that should be left empty are marked by stars on the boards below. A domino is drawn as a black rectangle.

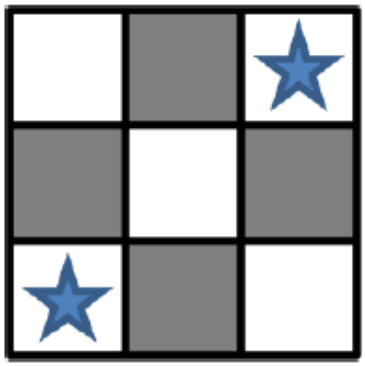
(a) Nina first tries a 2 by 2 board. Can a domino be placed on the board so that the lower left and upper right corners are empty?

(b) Nina next tries a 3 by 3 board. Here's a picture of a 3 by 3 board with a domino placed on it:



(c) What can you say about the colors of two squares that a domino covers on a chessboard?

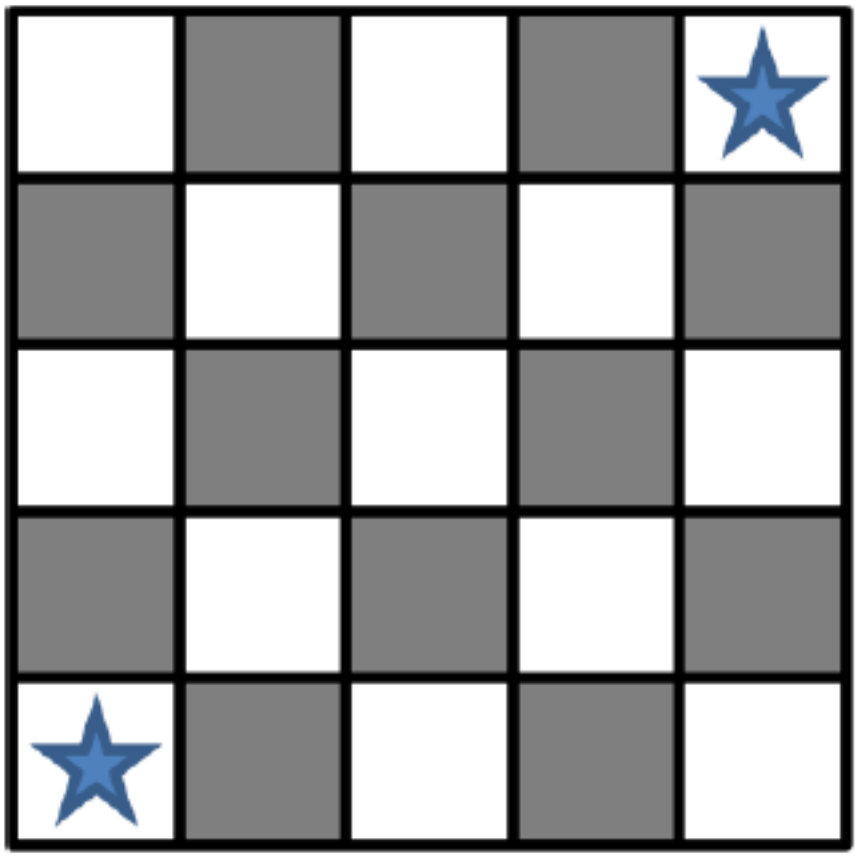
(d) Is there a way to lay down dominos to cover the 3×3 board so that only the lower left and upper right corners are empty? If yes, show how. If not, explain why not.



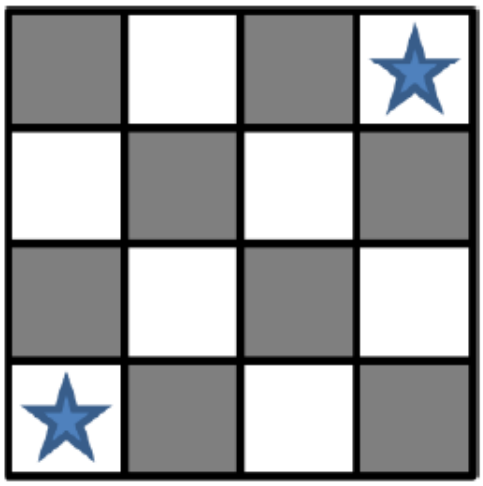
(e) Why did the 3 by 3 case not work?

(f) What is the biggest number of dominos you can place on the board above so that the lower left and upper right corners are left out?

(g) Would a 5 by 5 board work?



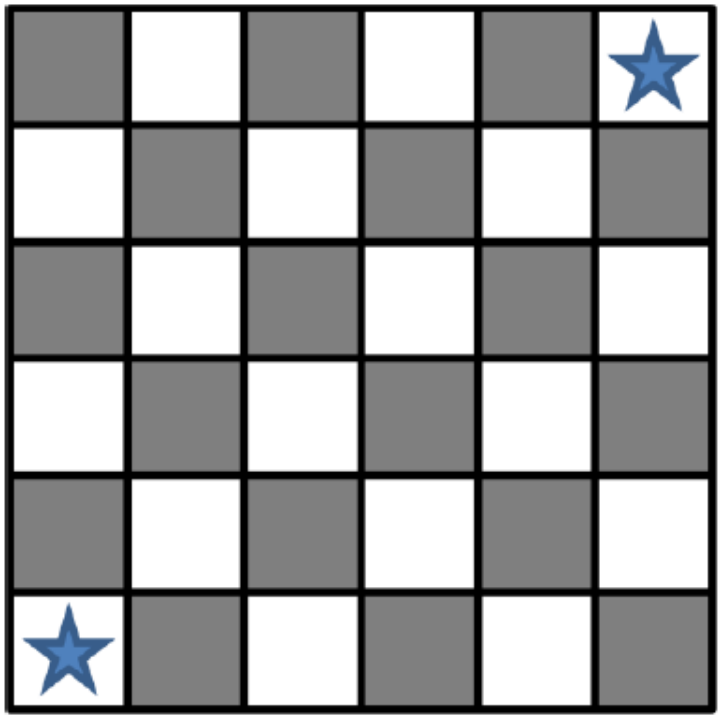
(h) Knowing that boards with odd numbers of squares don't work, Nina next tries a 4 by 4 board. Is there a way to cover all the squares except the two starred corners?



(i) What is the biggest number of dominos you can place on this board so that they do not cover the stars? How many squares are covered? How many squares are left uncovered?

(j) Why did the 4 by 4 case not work? (Hint: look at the colors of the corners that should be left empty)

(k) Would a 6 by 6 board work?



(l) Suppose you put as many dominos on the chessboard as you can (while leaving the two opposite corners empty).
How many white squares are left uncovered after that?
What about black squares?

(m) Nina now wants to design a chessboard where she can cover all the spaces except the lower left and upper right corners. She tries a 2 by 3 board and sees that it works.

Why does it work in this case?

Can you come up with any other size boards that work?

Note that next time we will have individual problem solving. You will be working on problems on your own without help from instructors. As homework, please go over binary numbers, Mayan notation for numbers, opposite statements, Sudoku and games we have learned today.