Answer Key

Problem 1   5 pts

A doctor prescribes a patient three pills and tells him to take one pill every hour. How long will the treatment take from the moment the patient takes the first pill?

2 hours

Problem 2   5 pts

Trains from San Francisco to Los Angeles depart every hour as well as the trains from Los Angeles to San Francisco. All the trains move at the same constant speed. Kate takes a train from San Francisco to Los Angeles to visit the Math Circle. During the trip, she sees the trains travelling from Los Angeles to San Francisco out of her window. What is the shortest time interval between the trains that Kate sees?

half an hour

Problem 5   10 pts

Use a compass and a ruler to construct a 60° angle. Do not use a protractor!

An equilateral triangle has $60°$ angles.

Problem 6   10 pts

You are facing the South Pole, your feet at the different sides of the International Date Line. What is the time difference between your left and right foot? Which foot is ahead of time of the other?

The right foot is 24 hours ahead of the left one.

Problem 7   10 pts

Two cyclists start out from two different cities at the same time riding towards each other along a straight highway. One of the cyclists rides at the speed of 10 mph, the other at 15 mph. At the moment of their start, a SuperFly starts from the helmet of one of them. Flying at the constant speed of 100 mph, the fly flies towards the second cyclist, meets the him, immediately turns around, flies to the first cyclist, meets the guy, turns around, flies to the second cyclist, and continues doing so until the cyclist meet 3 hours after their start. What distance does the fly cover?

300 miles

Problem 8 **20 pts**

a. $-1 \ (mod \ 7) \equiv 6$

b. $49 \ (mod \ 7) \equiv 0$

c. $101 \ (mod \ 7) \equiv 3$

d. $5 + 5 \ (mod \ 7) \equiv 3$

e. $3 \times 5 \ (mod \ 7) \equiv 1$

f. $1 \div 5 \ (mod \ 7) \equiv 3$

g. $3 \times 4 \ (mod \ 12) \equiv 0$

h. $3 \times 5 \ (mod \ 12) \equiv 3$

i. $-15 \ (mod \ 60) \equiv 45$

j. $360 - 58 \ (mod \ 60) \equiv 2$

Problem 9   **10 pts**

This year, July 1 is a Sunday. What day is July 31? Hint: $mod \ 7$ arithmetic can help.

    Tuesday

Problem 10   **10 pts**

Two pirates need to share a treasure composed of all sorts of things very different in nature and value, coins, jewelry, pieces of gold, precious stones, pearls, goblets, weapons, and so on. The pirates are very violent. Any suspicion of an unfair deal will lead to a brutal fight. How can the pirates share the treasure in such a way that will keep them both happy and prevent bloodshed?

    One pirate splits the treasure into two piles that seem to be of equal value to him. The other pirate chooses the pile that seems to be of greater value to him. Both are happy. The fifteen men on a dead man's chest problem does not arise. Yo-ho-ho and a bottle of rum!