LAMC Junior Circle **June 10, 2012**

Please print your name, first and last, followed by your school grade in the space below.

First Name Last Name Grade

The year-end problem solving session.

Pr #	1	2	3	4	5	6
Score	$\overline{5}$	$\overline{5}$	$\overline{10}$	$\overline{10}$	$\overline{10}$	$\overline{10}$

7	8	9	10	Total
$\overline{10}$	$\overline{20}$	$\overline{10}$	$\overline{10}$	$\overline{100}$

Problem 1 **5 pts**

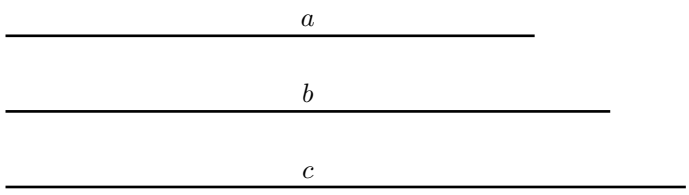
A doctor prescribes a patient three pills and tells him to take one pill every hour. How long will the treatment take from the moment the patient takes the first pill?

Problem 2 **5 pts**

Trains from San Francisco to Los Angeles depart every hour as well as the trains from Los Angeles to San Francisco. All the trains move at the same constant speed. Kate takes a train from San Francisco to Los Angeles to visit the Math Circle. During the trip, she sees the trains travelling from Los Angeles to San Francisco out of her window. What is the shortest time interval between the trains that Kate sees?

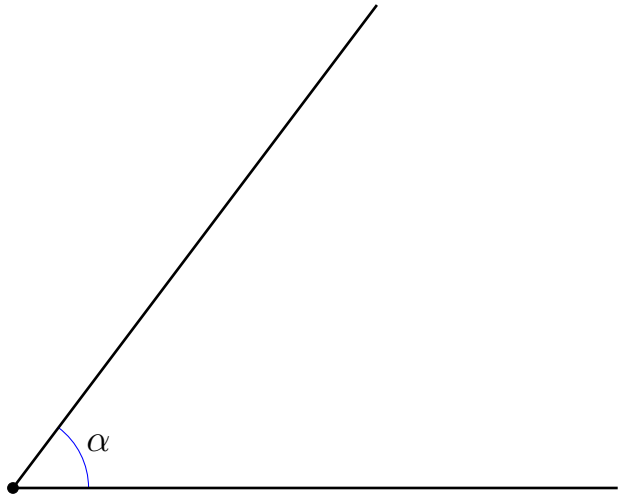
Problem 3 **10 pts**

Use a compass and a ruler to construct a triangle having the following sides.



Problem 4 **10 pts**

Use a compass and a ruler to construct an angle equal to the angle below and having a ray at the bottom of the page as a side. Do not use a protractor!



Problem 5 **10 pts**

Use a compass and a ruler to construct a 60° angle. Do not use a protractor!

Problem 6 **10 pts**

You are facing the South Pole, your feet at the different sides of the International Date Line. What is the time difference between your left and right foot? Which foot is ahead of time of the other?

Problem 7 **10 pts**

Two cyclists start out from two different cities at the same time riding towards each other along a straight highway. One of the cyclists rides at the speed of 10 mph, the other at 15 mph. At the moment of their start, a SuperFly starts from the helmet of one of them. Flying at the constant speed of 100 mph, the fly flies towards the second cyclist, meets the him, immediately turns around, flies to the first cyclist, meets the guy, turns around, flies to the second cyclist, and continues doing so until the cyclist meet 3 hours after their start. What distance does the fly cover?

Problem 8 **20 pts**

a. $-1 \ (mod\ 7) \equiv$

b. $49 \ (mod\ 7) \equiv$

c. $101 \ (mod\ 7) \equiv$

d. $5 + 5 \ (mod\ 7) \equiv$

e. $3 \times 5 \ (mod\ 7) \equiv$

f. $1 \div 5 \ (mod\ 7) \equiv$

g. $3 \times 4 \ (mod\ 12) \equiv$

h. $3 \times 5 \ (mod\ 12) \equiv$

i. $-15 \ (mod\ 60) \equiv$

j. $360 - 58 \ (mod \ 60) \equiv$

Problem 9 **10 pts**

This year, July 1 is a Sunday. What day is July 31? Hint: *mod* 7 arithmetic can help.

Problem 10 **10 pts**

Two pirates need to share a treasure composed of all sorts of things very different in nature and value, coins, jewelry, pieces of gold, precious stones, pearls, goblets, weapons, and so on. The pirates are very violent. Any suspicion of an unfair deal will lead to a brutal fight. How can the pirates share the treasure in such a way that will keep them both happy and prevent bloodshed?