A LEVER PLAYING FIELD 2: NEVER SAY LEVER

MATH CIRCLE (BEGINNERS) 05/27/2012

The Law of the Lever, first proved by the Greek mathematician Archimedes, describes the situation in which a scale/balance/lever will be balanced.

Consider a lever that has a weight of W_1, at a distance D_1 to the left of the fulcrum, and a weight of W_2 at distance D_2 to the right of the fulcrum (as pictured). Then the Law of the Lever states that the two weights will be balanced if, and only if

$$W_1 D_1 = W_2 D_2.$$

More generally, if there are multiple weights on each side, then the *sum* of the weight$\times$distance values on the left side, must equal the sum of the weight$\times$distance values on the right side. For example if there were weights W_1 and W_2 on the left side at distances D_1 and D_2 respectively; and weight W_3 on the right side at distance D_3, then the scale will be balanced if and only if

$$W_1 D_1 + W_2 D_2 = W_3 D_3.$$

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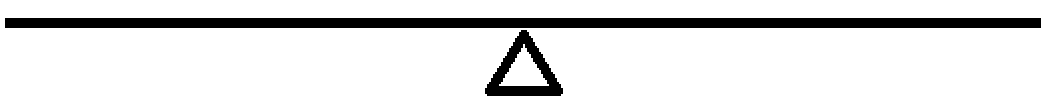
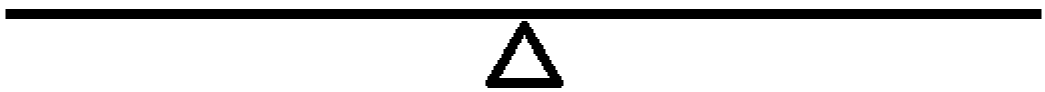
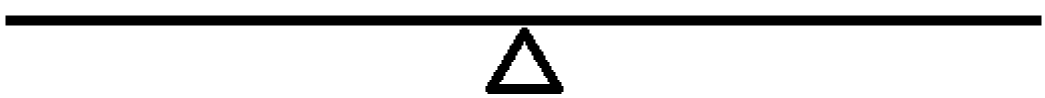
(1) You have a 3-lb. weight and a 5-lb. weight. On the lever below, place the weights so that the scale will be balanced. **In this and all subsequent problems, label the weights you use (here 3 lb, 5 lb), and indicate the distance of each one from the fulcrum.** (Hint: There is not just one possible answer, although there is arguably one that is "simplest".)



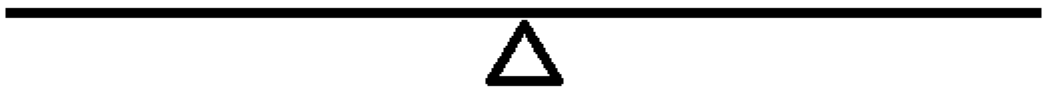
(2) You have a 1-lb. weight, a 2-lb. weight, and a 3-lb. weight. Place them on the scale so that it will be balanced, being sure to label the weights and their distances. (Again, there are multiple possible answers.)



(3) Place a 10-lb. weight on the following scale, so that it balances with a weight which is 15 ft. from the fulcrum on the right side. You choose the weight on the right side, and the distance of the 10-lb. weight.

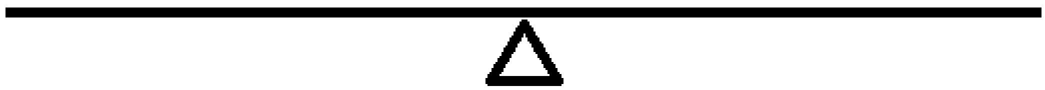


(4) Now solve problem (3) again, but give a different answer:



(5) This scale will use a 3kg weight, a 7kg weight, and a 9kg weight. The distances of the weights should be 2m, 3m, and 3m (but not necessarily in that order, and I'm not telling you on which side of the balance each distance should be!) Can you figure out where to place the weights, according to those rules, to balance the scale?

(6) The following balance is not balanced. It currently has a weight of 2kg suspended 1.6m to the left of the fulcrum, and a weight of 1kg suspended 6.4m to the right. **(Draw and label these weights and distances.)** The total length of the bar is 20m (10 on the left, 10 on the right). What is the weight of the smallest possible weight you could add to balance the scale, and where should you place it?

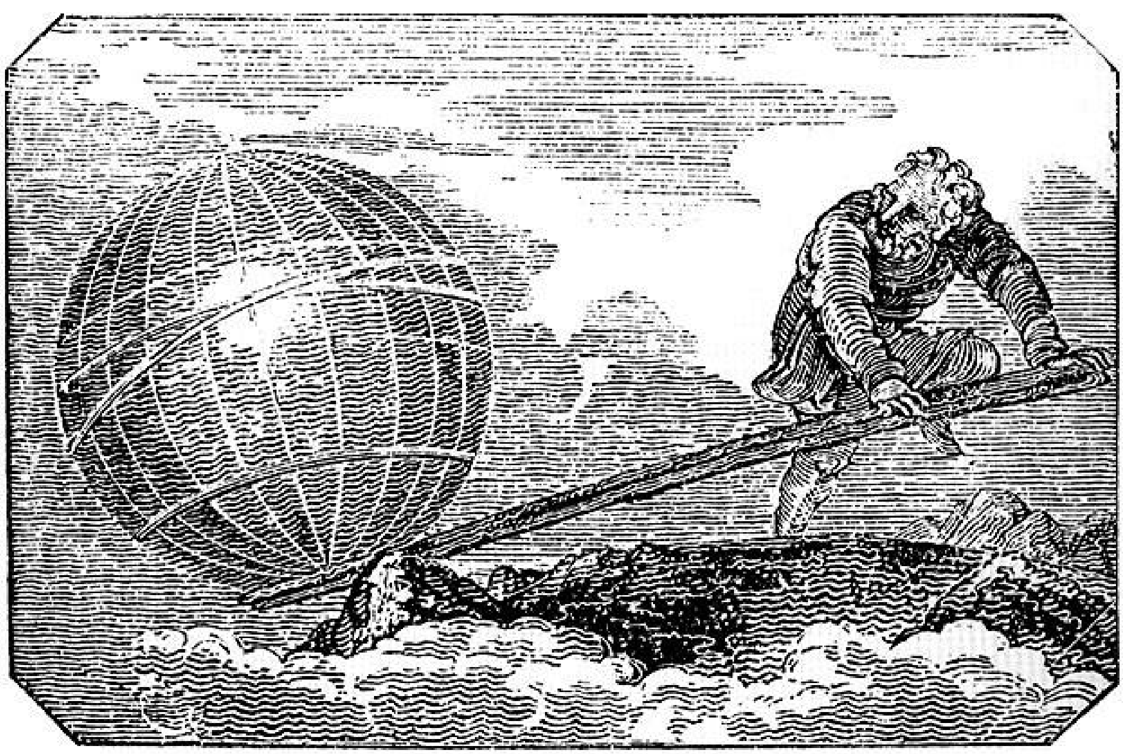


(7) The following bar has a weight of 15 lbs. at the far left and and 3 lbs. at the far right (draw them!). Where along the bar should you place the fulcrum so that it will balance? (Draw and label!) You can imagine that the bar itself weighs nothing.

(8) In the previous problem you were supposed to imagine that the bar weighed nothing. Of course in reality a bar WILL weigh something. The bar below is actually rather heavy; it weighs 4 lbs. The distance from the fulcrum to the left end of the bar is 2 feet, and I have hung a 1 lb. weight at the left end. There is nothing hanging on the right side.

How long is the bar? (Picture may not be to scale.) (Hint: Where is a bar's center of gravity, in general?)

(9) Archimedes reportedly once boasted about the usefulness of levers, saying, "Give me the place to stand, and I shall move the earth." (See picture—not to scale.)



(a) About how much mass ("weight") does the Earth have, in either lbs. or kilograms? (Use any knowledge you, your neighbors, your assistant, etc. have to formulate a best-guess estimate.)

(b) About how many lbs. or kilograms (use same as above) of downward force can a human generate? (Use any knowledge you, your neighbors, your assistant, etc. have to formulate a best-guess estimate.)

(c) Given your answers to (a) and (b), about how long a lever would Archimedes need in order to move (or just balance) the world, assuming he found a place to stand, and the fulcrum were placed 1 meter from where the world rested on the lever? (You can imagine the lever itself is weightless, like in Problem (7).)