COMBINATION III

MATH CIRCLE (INTERMEDIATE) 5/27/2012

1) How many ten-digit numbers have the sum of their digits equal to

a) 2;

b) 3;

c) 4?

2) How many ways are there to divide a deck of 36 cards, including 4 aces, into halves so that each half contains exactly 2 aces?

3) We have 10 u's and 6 r's. How many words can we make using these letters?

4) We have a 11×7 lattice.

a) In how many ways can we go from the bottom left corner to the top right corner?

b) Is there a connection between this question and the previous question?

5) Expand the following expressions:

a) $(x+y)^2$

b) $(x+y)^3$

c) $(x+y)^4$

Challenge 1) a) Find coefficient of $x^k y^{n-k}$ in the expansion of $(x+y)^n$.

b) Substitute $x=1$ and $y=-1$ in above and deduce that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - + \cdots + (-1)^n \binom{n}{n} = 0$$

Challenge 2) Each side of a boat must be occupied by exactly 4 rowers. How many ways are there to choose a rowing team for the boat if we have 31 candidates, ten of whom want to be on the left side of the boat, twelve on the right side, and the other nine can sit on the either side?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg "Mathematical Circles (Russian Experience)"
- Previous UCLA Math Circle notes

Warm up 1) Joe has 3 shirts, 2 t-shirts, 4 pants, 3 pairs of sandals and 2 pairs of shoes. In how many ways can he dress up given that he does not want to wear a shirt and a pair of sandals at the same time?

Warm up 2) How many different words can you make by rearranging the alphabets of "MATHEMATICS" ?