

LAMC Junior Circle

April 8, 2012

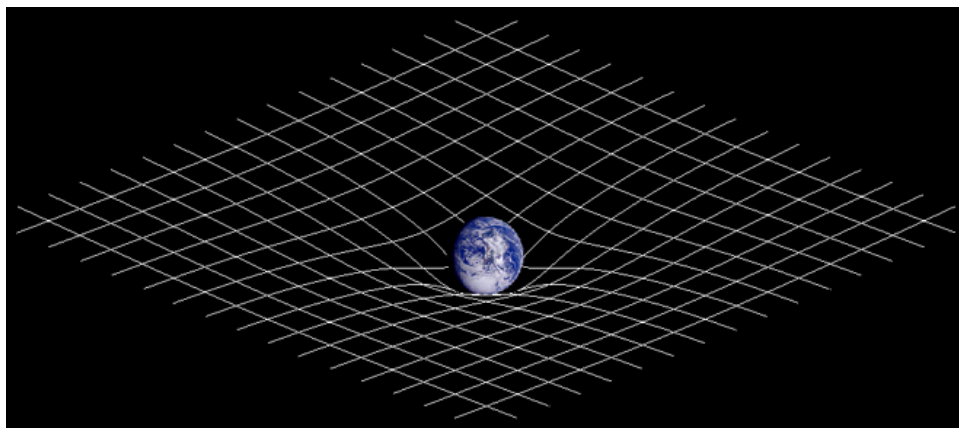
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**Geometry, the Art of Measuring Earth. A Line and a
Straight Line.**

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Geometry is a composite word. In the ancient Greek, *geo* meant *earth* and *metria* meant *measure*. Geometry was born as an art of measuring land lots. It proceeded to become a science that explains, among other things, gravity as curvature of warped space-time.



The Earth bending space and time around it.

Euclid, a Greek mathematician who lived about 300 BC in the city of Alexandria, is often referred to as the “Father of Geometry”. He is the author of the first geometry textbook on this planet, the “Elements”. The “Elements” is not only the first textbook on Earth, it is also the most successful one. It was in circulation from the moment it appeared to the beginning of the 20th century, over 2,200 years altogether. Euclid didn’t discover most of the mathematical statements contained in the “Elements”. He rather presented them in a logical and coherent way, including a system of rigorous proofs, that remains the basis of mathematics 23 centuries later.



Euclid¹

¹This is an artist’s rendering. No real portrait has reached our times.

An *axiom* is a statement taken for granted as self-evident and not needed to be proved. All other math statements are derived from axioms using formal logic. A *proof* is a way to convince your peers (without resorting to violence).

Axiom 1 (*Euclid*) *A point is that which has no part.*

Problem 1 *Prove that a point has no length, widths, or height.*

Problem 2 *Draw a point on the left-hand side of the space below. Imagine that you explore the drawing using some significant magnification, a powerful lens or better a microscope. Draw the picture you would see on the right-hand side of the space below.*



Problem 3 *Imagine that you look at an ideal point, an object with no length, height, or width, through a microscope. Would the picture change if you increase magnification?*

Since a point has no size in any direction, it is sometimes called a *zero-dimensional space*.

Axiom 2 (*Euclid*) *A line is breadthless length.*

A line (not necessarily straight) has no height or width, only length. As any other geometric object, a line is made of points. Since, out of all dimensions, a line has only length, we call it a one-dimensional (1D) object.

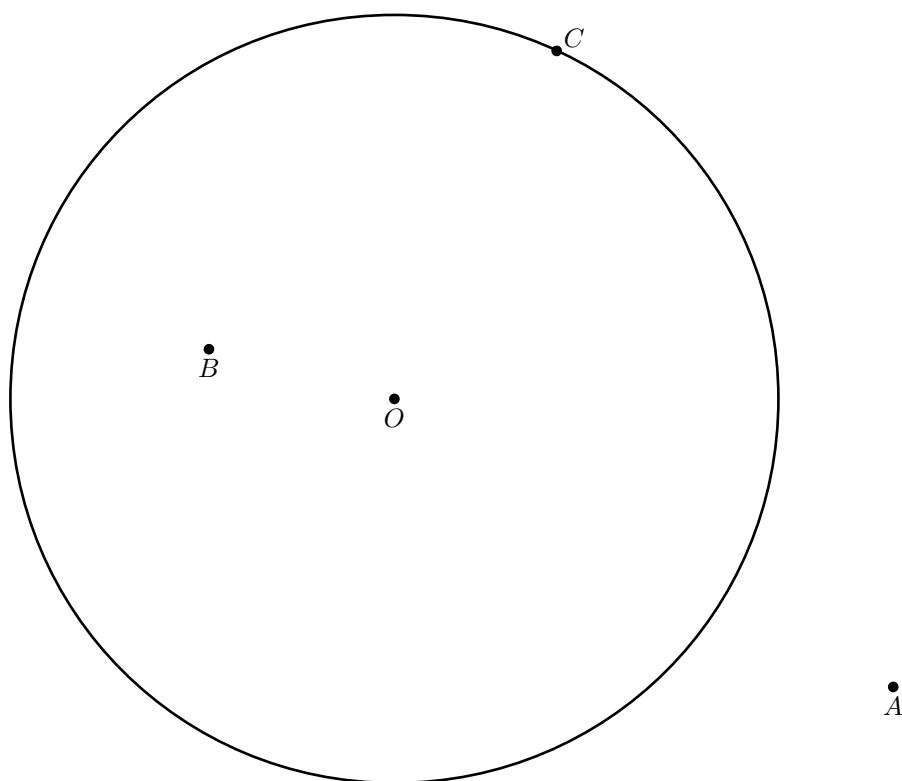


Problem 4 *Draw a line on the left-hand side of the space below. Imagine that you explore the drawing using some significant magnification, a powerful lens or microscope. Draw the picture you would see on the right-hand side of the space below.*



A *circumference* is the set of all the points in the plane having an equal distance, called the *radius*, from a special point, called the *center*.

Problem 5 *Measure in inches the radius of the following circumference.*



Measure the distances $|OA|$, $|OB|$, and $|OC|$ with a ruler.

- *Which distance equals the radius? What point lies on the circumference?*

- *Which distance is less than the radius? What point lies inside the circumference?*

- *Which distance is greater than the radius? What point lies outside of the circumference?*

- *Pick a point on the circumference different from the point C. Tell the distance from the circumference center to the point without measuring it. Recall the definition of the circumference if needed.*

- *Pick a point different from the point A outside of the circumference. Is the distance from the circumference center to the point greater or less than the radius? Check the answer by a direct measurement, comparing the distance to the 2" mark on the ruler.*

- *Pick a point different from the point B inside the circumference. Is the distance from the circumference center to the point greater or less than the radius? Check the answer by a direct measurement.*

Problem 6 *Is a circumference a line?*

A *circle* of *radius* r is the set of all the points in the plane such that their distance to a special point called the *center* is less than or equals to r .

Problem 7 *What is the difference between a circle and a circumference?*

Problem 8 *Take another look at the picture in Problem 5. Which of the points A , B , C , O belong to the circumference? To the circle?*

The circumference:

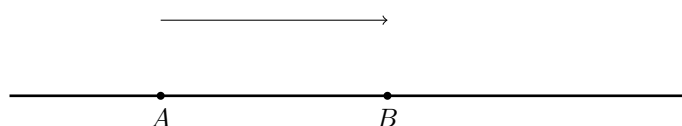
The circle:

Problem 9 *Suppose that you have to tend after a goat. You hammer a stake in the center of a flat meadow, tie up the goat to the stake with a rope of length $r = 5$ feet (5') and let it graze. What figure do you get when the animal eats up all the grass it can reach?*

Problem 10 *Is a circle a line?*

Axiom 3 (Euclid) *A straight line is a line that lies evenly with the points on itself.*

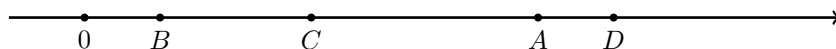
What Euclid probably meant was that you can move any point of a straight line to the position of any other point by sliding the line along itself. Such a move is called a *translation*.



A *symmetry* of a geometric object is a transformation that moves its points around, but preserves the object as a whole. Euclid states that, unlike a generic line, a straight line is a highly symmetric object. Every translation of a straight line is a symmetry! It is this huge amount of symmetries that allows us to add and subtract numbers!

Problem 11

• Using a compass, mark the points $B + C$ and $D - A$ on the following number line.



• Put the correct sign, $>$, $<$, or $=$, in the box between the numbers.

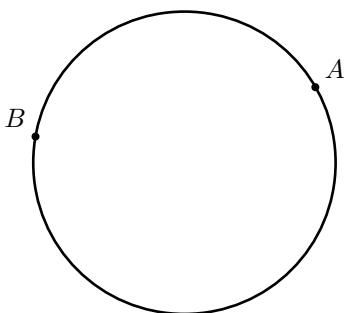
$$B + C \square A \quad B \square 0 \quad B \square D - A$$

- Using a ruler marked with centimeters, find the numerical values of the numbers A , B , C , D , $B + C$, and $D - A$. Having obtained the numerical data, check correctness of the above geometric equalities and inequalities.

$$A = \quad B = \quad C =$$

$$D = \quad B + C = \quad D - A =$$

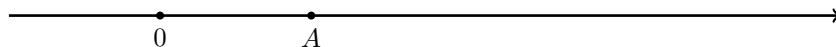
Problem 12 Can we slide point A to the position of point B by a move that preserves the following line?



Can we do it for any two points on a circumference?

Question 1 *It seems that, according to Axiom 3, a circumference is a straight line. How can we clarify the situation?*

Problem 13 *Use a compass to find the points $2A$ and $3A$ on the line below.*



Let us think of multiplication as of stretching the number line. This operation moves the points of the line, but preserves the line as a whole, so it also is a symmetry, called *dilation*. If we take both dilations and translations into account, then Axiom 3 defines nothing but a straight line!

Question 2 *Do you think Euclid really meant all that?*

Problem 14 *Imagine that you are an ancient Greek. You need to multiply some length by five. Can you do it using nothing but a rope? Draw a picture that shows how.*

Problem 15 *Use a rope to find 10×2^6 .*

Homework Problem 1 *How many symmetries has a square?*