AMC PROBLEMS WITH COOL SOLUTIONS!
SELECTED BY ALI GUREL

(1) (AMC10A-2006-20) Six distinct positive integers are randomly chosen between 1 and 2006, inclusive. What is the probability that some pair of these integers has a difference that is a multiple of 5?

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{4}{5}$ (E) 1

(2) (AMC10B-2002-12) For which of the following values of k does the equation $\frac{x-1}{x-2}=\frac{x-k}{x-6}$ have no solution for x?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

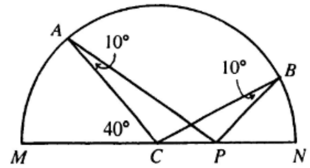
(3) (AMC10-2000-22) One morning each member of Angela's family drank an 8-ounce mixture of coffee with milk. The amounts of coffee and milk varied from cup to cup, but were never zero. Angela drank a quarter of the total amount of milk and a sixth of the total amount of coffee. How many people are in the family?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

(4) (AMC12-2001-16) A spider has one sock and one shoe for each of its eight legs. In how many different orders can the spider put on its socks and shoes, assuming that, on each leg, the sock must be put on before the shoe?

(A)$8!$ (B)$2^8 \cdot 8!$ (C)$(8!)^2$ (D)$\frac{16!}{2^8}$ (E)$16!$

(5) (AHSME-1983-30) Distinct points A and B are on a semicircle with diameter MN and center C. The point P is on CN and $\widehat{CAP}=\widehat{CBP}=10^\circ$. If $\overset{\frown}{MA}=40^\circ$, then $\overset{\frown}{BN}$ equals



(A) 10° (B) 15° (C) 20° (D) 25° (E) 30°

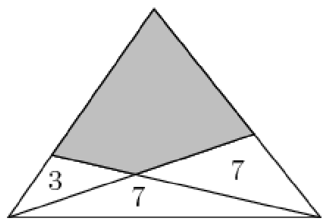
(6) (AHSME-1992-17) The two-digit integers from 19 to 92 are written consecutively to form the large integer

$$N = 19202122\ldots909192.$$

If 3^k is the highest power of 3 that is a factor of N, then $k =$

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

(7) (AMC10B-2006-23) A triangle is partitioned into three triangles and a quadrilateral by drawing two lines from vertices to their opposite sides. The areas of the three triangles are 3, 7, and 7 as shown. What is the area of the shaded quadrilateral?



(A) 15 (B) 17 (C) $\frac{35}{2}$ (D) 18 (E) $\frac{55}{3}$

(8) (AHSME-1988-18) At the end of a proffessional bowling tournament, the top 5 bowlers have a play-off. First #5 bowls #4. The loser receives 5^{th} prize and the winner bowls #3 in another game. The loser of this game receives 4^{th} prize and the winner bowls #2. The loser of this game receives 3^{rd} prize and the winner bowls #1. The winner of this game gets 1^{st} prize and the loser gets 2^{nd} prize. In how many orders can bowlers #1 through #5 receive the prizes?

(A) 10 (B) 16 (C) 24 (D) 120 (E) none of these

(9) (AMC12-2000-20) If x, y, and z are positive numbers satisfying

$$x + 1/y = 4, \quad y + 1/z = 1, \quad \text{and} \quad z + 1/x = 7/3,$$

then what is the value of xyz?

(A) 2/3 (B) 1 (C) 4/3 (D) 2 (E) 7/3

(10) (AHSME-1991-20) The sum of the real x such that

$$(2^x - 4)^3 + (4^x - 2)^3 = (4^x + 2^x - 6)^3 \text{ is}$$

(A) 3/2 (B) 2 (C) 5/2 (D) 3 (E) 7/2

(11) (AMC12B-2006-18) An object in the plane moves from one lattice point to another. At each step, the object may move one unit to the right, one unit to the left, one unit up, or one unit down. If the object starts at the origin and takes a ten-step path, how many different points could be the final point?

(A) 120 (B) 121 (C) 221 (D) 230 (E) 231

(12) (AMC10A-2010-22) Eight points are chosen on a circle, and chords are drawn connecting every pair of points. No three chords intersect in a single point inside the circle. How many triangles with all three vertices in the interior of the circle are created?

(A) 28 (B) 56 (C) 70 (D) 84 (E) 140

(13) (AMC10A-2008-23) Two subsets of the set $S = \{a, b, c, d, e\}$ are to be chosen so that their union is S and their intersection contains exactly two elements. In how many ways can this be done, assuming that the order in which the subsets are chosen does not matter?

(A) 20 (B) 40 (C) 60 (D) 160 (E) 320

(14) (AMC10-2000-25) In year N, the 300^{th} day of the year is a Tuesday. In year $N + 1$, the 200^{th} day is also a Tuesday. On what day of the week did the 100^{th} day of year $N - 1$ occur?

(A) Thursday (B) Friday (C) Saturday (D) Sunday (E) Monday

(15) (AMC12B-2004-22) The square

50	b	c
d	e	f
g	h	2

is a multiplicative magic square. That is, the product of the numbers in each row, column, and diagonal is the same. If all the entries are positive integers, what is the sum of the possible values of g?

(A) 10 **(B)** 25 **(C)** 35 **(D)** 62 **(E)** 136