To perform these operations successfully one must know the addition and multiplication tables for numbers less than the base of the system—that is, for one-digit numbers. For the decimal system, we have learned it early and well.

Exercise 5. Write down these tables for systems with bases 2, 3, 4, and 5.

Exercise 6. Calculate a) $1100_2 + 1101_2$; b) $201_3 \cdot 102_3$.

<u>**For teachers.**</u> We explained here very briefly how to add and multiply the numbers in any number base system. In a real session this would take more time. Of course, the goal of this work is not speed or accuracy in computations written in another number base system. An examination of and some practice in the addition and multiplication algorithms written in systems other than base 10 can lead to a deeper understanding of these algorithms.

Now we describe an effective algorithm for converting from one number system to another. It differs from the one we already know, because now the representation of a number will appear digit by digit from right to left rather than from left to right. The last digit is just the remainder when the number is divided by the base of the new system. The second digit can be found as follows: we take the quotient from the previous calculation and find the remainder when the quotient is divided by the base of the new system. Then we proceed in exactly the same way until we complete the representation.

Example. Let us convert the number 250_{10} to the base 8 ("octal") system:

$$\begin{aligned} 250 &= 31 \cdot 8 + \mathbf{2}, \\ 31 &= 3 \cdot 8 + \mathbf{7}, \\ 3 &= 0 \cdot 8 + \mathbf{3}. \end{aligned}$$

Thus, $250_{10} = 372_8$.

Solve →

Exercise 7. Convert to the base 7 system the numbers a) 1000_{10}; b) 532_8.

In conclusion we submit a few more interesting problems.

Solve →

Problem 1. A teacher sees on the blackboard the example $3 \cdot 4 = 10$. About to wipe it away, she checks if perhaps it is written in another number base system. Could this thought have been right?

Solve →

Problem 2. Does there exist a number system where the following equalities are true simultaneously:

a) $3 + 4 = 10$ and $3 \cdot 4 = 15$;

b) $2 + 3 = 5$ and $2 \cdot 3 = 11$?

Solve →

Problem 3. State and prove a condition (involving the representation of a number) which allows us to determine whether the number is odd or even

a) in the base 3 system;

b) in the base n system.

Solve →

Problem 4. A blackboard bears a half-erased mathematical calculation exercise:

$$\begin{array}{r} 2\ 3\ ?\ 5\ ? \\ +\quad 1\ ?\ 6\ 4\ 2 \\ \hline 4\ 2\ 4\ 2\ 3 \end{array}$$

Find out what number system the calculation was performed in and what the summands were.