LAMC Early Elementary Circle October 30, 2011

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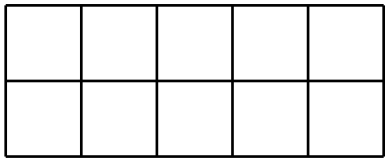
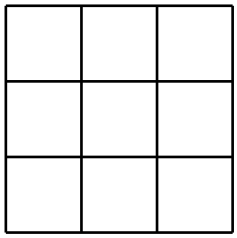
Young diagrams

Part 2

Problem 1 *The Jones family has a garden, a, and a vegetable garden, b. The length of a side of the small squares on the picture below is 1 m (meter). What is the total area of the Jones family gardens in square meters (m^2)? Use skip-counting by rows to figure out, then skip-count by columns to double-check.*



a



b

Definition 1 *Let x and y be some (positive integral) numbers. Then*

$$x \times y = \underbrace{x + x + \ldots + x}_{y \text{ times}}$$

The numbers x and y are called multiples, the result is called their product.

The area of the Jones family garden a from the above example is $2 \times 5 = 5 \times 2 = 10$. Is it a coincidence that changing the order of multiples does not change the product? Let us see.

Problem 2 *Using Young diagrams, compute*

$3 \times 8 =$ $\qquad\qquad$ $8 \times 3 =$

Problem 3 *Prove that $x \times y = y \times x$ for any (positive integral) numbers x and y. Hint: does the area of a rectangular garden change when we skip-count it first by rows and then by columns?*

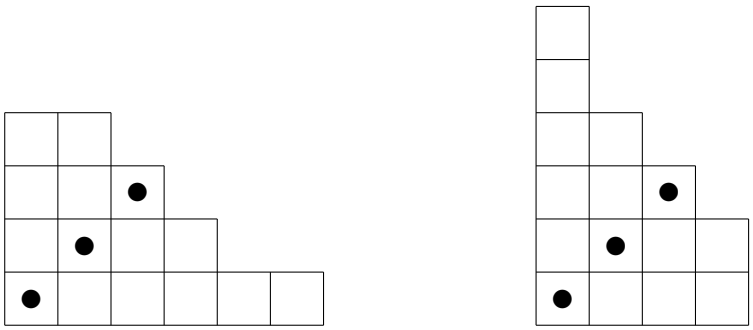
The above property of multiplication is called *commutativity*. Multiplication is a commutative operation. This means that the order of multiples does not matter, $x \times y$ always equals $y \times x$.

Problem 4 *Is addition commutative? Is subtraction?*

Problem 5 *Thomas, Daniella and Gregory have 18 candies to share. Can each of them have an equal number of candies? What is the number? Using the grid below, draw the corresponding Young diagram to figure out.*

Problem 6 *This time, Thomas, Daniella and Gregory have 19 candies to share. Can each of them have an equal number of candies? What is the best they can do? Using the grid below, draw the corresponding Young diagram to figure out.*

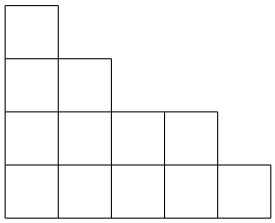
As we have seen proving commutativity of multiplication, the number of boxes in a Young diagram does not change when we flip it over the diagonal, like below.

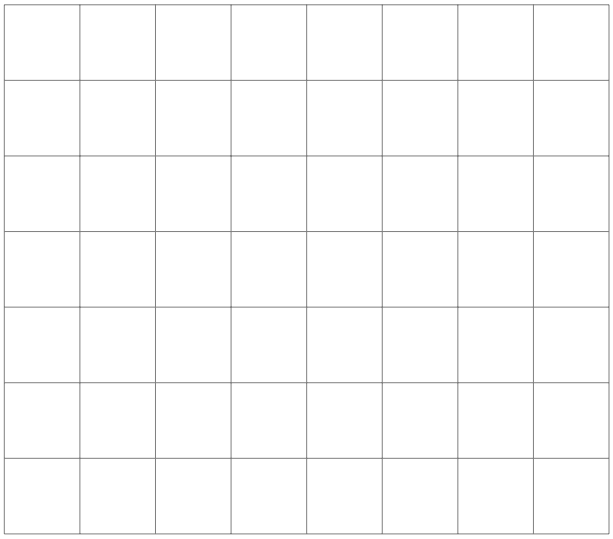


This operation is called *conjugation*. The diagrams are called conjugate to each other.

Problem 7 *Write down the conjugate partitions of the number 15 corresponding to the above pair of conjugate diagrams. (Recall that a part of a partition is the number of apartments on the corresponding floor.)*

Problem 8 *Draw the Young diagram conjugate to the one below. Write down both partitions.*





Problem 9 *Find all the partitions of 10 into parts not exceeding 2. In other words, count all the houses with 10 apartments and with no more than 2 apartments on every floor. Hint: a partition with parts not exceeding 2 is conjugate to a partition with no more than 2 parts, see below.*