

PIGEON HOLE PRINCIPLE I

MATH CIRCLE (INTERMEDIATE) 11/13/2011

1) A bag contains 5 black beads and 5 white beads. How many beads do you need to draw (without looking) to ensure you have two beads of the same color?

2) One million pine trees grow in a forest. It is known that no pine tree has more than 600,000 pine needles on it. Show that two pine trees in the forest must have the same number of pine needles.

3) Twenty-five crates of apples are delivered to a store. Each crate contains one of three types of apples. Show that among the 25 crates there are at least 9 containing the same type of apples.

4) Given 12 integers, show that two of them can be chosen whose difference is divisible by 11.

5) Show that in any group of 5 people, there are two who have an identical number of friends within the group.

6) Ten students solved a total of 35 problems in a math olympiad. Each problem was solved by exactly one student. There is at least one student who solved exactly one problem, at least one student who solved exactly two problems, and at least one student who solved exactly three problems. Prove that there is at least one student who solved at least five problems.

7) What is the largest number of kings which can be placed on a chessboard so that no two of them attack each other?

8) What is the largest number of squares on an 8×8 chessboard which can be colored green, so that in any arrangement of three squares (a “tromino”) at least one square is not colored green?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg “Mathematical Circles (Russian Experience)”
- Previous UCLA Math Circle notes