

EGYPTIAN MULTIPLICATION

JUNIOR CIRCLE 10/31/2011

Ancient Egyptians had an interesting method for multiplying two numbers. Suppose that you have to multiply two numbers (e.g., 21 and 18). The basic operation for them was multiplying a number by 2. (In other words, adding a number to itself). They reduced all other multiplication problems to it. Here is how they would start multiplying 21 by 18 (in modern notation):

$$\begin{array}{r} 21 \quad 18 \\ \hline 1 \quad 18 \\ 2 \quad 36 \\ 4 \quad 72 \\ 8 \quad 144 \\ 16 \quad 288 \end{array}$$

Here is what they did to complete the multiplication

- (1) Below the first number (in this case, 21), they would write all of the powers of 2 that are smaller or equal to the first number.
- (2) In the second column, they would keep doubling the second number (in this case, 18).
- (3) After that, they would represent the first number as the sum of the powers of 2 (so that each of the powers of 2 is used at most once).

For example, if the first number is 21, they would find

$$21 = 16 + 4 + 1.$$

After that, they would mark those rows where these powers of 2 are present in the left column. (In our example, the first, the third and the fifth rows are marked).

Finally, all there is to do at this point is to add the marked numbers in the second column:

$$\begin{array}{r} \\ \\ + \\ \hline 3 \end{array}$$

Thus, the result of the multiplication is 378.

Multiply 13 by 22 using Egyptian multiplication.

The goal of our session today is to understand how and why the Egyptian's method of multiplication worked.

- (1) Why did the Egyptian's use rows that are not marked? (E.g. with numbers 36, 144 in the second column in our example.)

- (2) Would

$$38 = 2 + 4 + 16 + 16$$

be a representation of 38 that Egyptians would use in a multiplication problem? Why or why not?

- (3) Represent each of the numbers below as a sum of powers of 2, where each of the powers is used at most once.

(a) $9 =$

(b) $14 =$

(c) $23 =$

(d) $44 =$

-
- (e) Is it easier to start by finding the smallest or the largest power? Why?
- (f) Describe the best way to find the representation of a number as the sum of powers of 2.
- (4) Explain how each number in the second column is obtained from the number in the first column and the same row and the second number we are multiplying (18 in our example above).
E.g. How do you get
- 18 from 1 and 18?
 - 144 from 8 and 18?

(5) Using what you noticed in question 4 do the following:

(a) Rewrite each term in the sum: $18 + 72 + 144$ using the result from question 4:

- $18 =$

- $72 =$

- $288 =$

(b) Rewrite the whole sum:

- $18 + 72 + 288 =$

(c) What do you notice? Can you simplify this expression by factoring out 18?

(6) Multiply the following numbers using Egyptian Multiplication:

(a) 13×41

(b) 41×13

(7) Given two numbers, which one (smaller or larger) will you use as the first number in Egyptian Multiplication? Why?

(8) Explain how Egyptian Multiplication works.

(9) With a partner, have a race to see who can multiply numbers faster. One of you must use Egyptian Multiplication and the other must use regular, long multiplication. Race 4 times switching the type of multiplication you do. Show your work below:

(a) 25×31

(b) 38×45

(c) 12×63

(d) 17×52

(10) Is it easier to use regular, long multiplication or Egyptian Multiplication? Which one is faster?

(11) Which one requires you to remember more facts? Which facts must you remember?