

DIVISIBILITY I

MATH CIRCLE (INTERMEDIATE) 10/30/2011

1) Given two different prime numbers p and q , find the number of different divisors of the number a) pq ; b) p^2q ; c) p^2q^2 ; d) p^nq^m .

2) Prove that the product of any three consecutive natural numbers is divisible by six.

3) Find the smallest natural number n such that $n!$ is divisible by 990.

4) For some number n , can the number $n!$ have exactly five zeros at the end of its decimal representation?

5) Find all solutions in the natural numbers to the equation $x^2 - y^2 = 31$.

6) Prove that any two natural numbers a and b satisfy the equation
$$\gcd(a, b) \cdot \text{lcm}(a, b) = ab.$$

Challenge 1) Prove that if a number has an odd number of divisors, then it is a perfect square.

Challenge 2) Can a number written with one hundred 0's, one hundred 1's, and one hundred 2's be a perfect square?

Problems are taken from:

- D. Fomin, S. Genkin, I. Itenberg "Mathematical Circles (Russian Experience)"
- Previous UCLA Math Circle notes