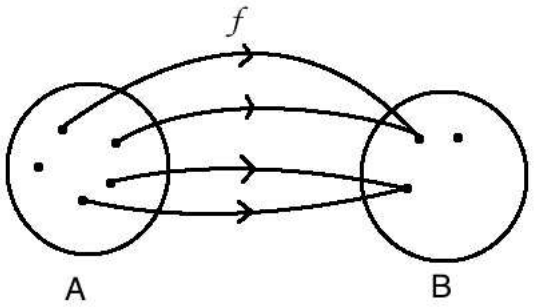
SETS AND FUNCTIONS II

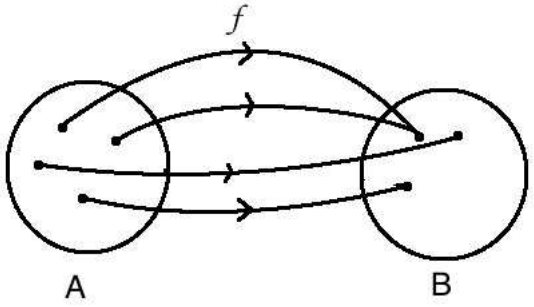
JUNIOR CIRCLE 10/16/2011

(1) Use the pictures to answer the questions below.

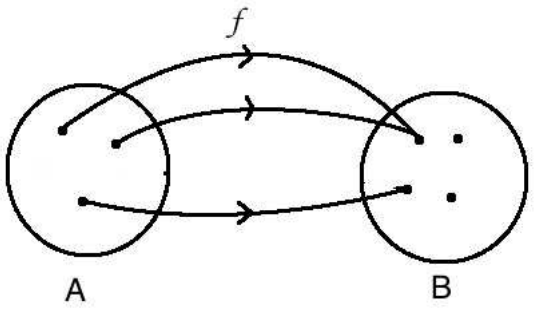
(a) Determine whether this function is defined. If yes, decide whether it is $1-1$ and whether it is onto.



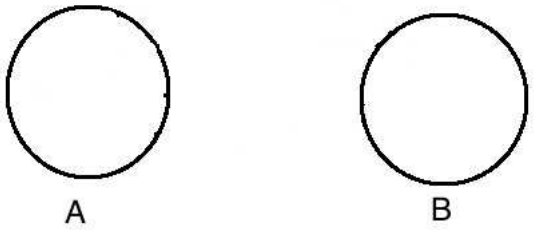
(b) Determine whether this function is defined. If yes, decide whether it is $1-1$ and whether it is onto.



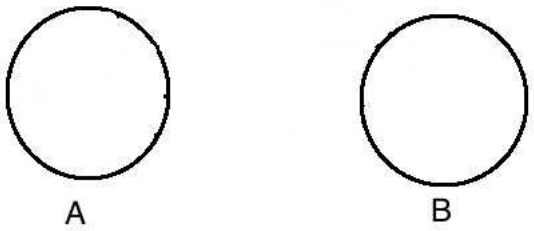
(c) Determine whether this function is defined. If yes, decide whether it is $1-1$ and whether it is onto.



(d) Make a picture showing a function that is onto, but not $1-1$.



(e) Make a picture showing a function that is $1-1$ and onto.



What can you say about the number of elements in both sets?

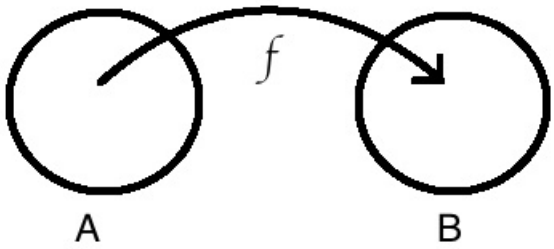
(2) Let A and B be two sets each consisting of n elements (where n is a finite number). Explain how we can construct a function $f : A \to B$ such that
- f is onto and
- f is $1-1$.

(*Hint*: number the elements of each set first. Make a picture.

Review:

A *function* from a set A to a set B is a rule that assigns to each element in A an element in B. The rule can be expressed in words, as a picture, by a table of values or as a formula.

- Remember that function is a rule we must follow. It can also be looked at as an assignment of an element in B for every element in A.
- The notation $f : A \to B$, which reads "a function from the set A to the set B" is used to denote a function from A to B.



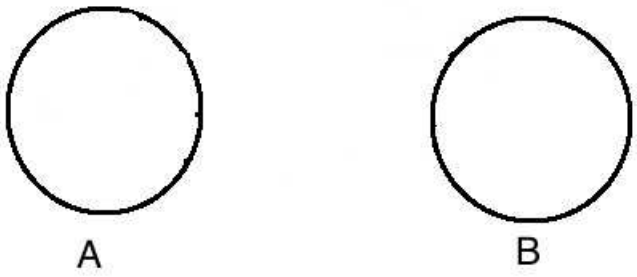
Exercise 1. Which set (A or B) do we look at when we are checking if a function is defined? Explain why. You may draw a picture, but a written answer is necessary.

Exercise 2. What does it mean for a function to be onto? Which set (A or B) do we look at when we are checking if a function is onto? Explain why.

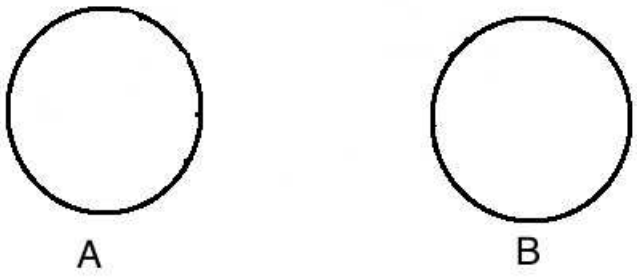
Exercise 3. What does it mean for a function to be $1-1$?

Exercise 4. Suppose that A and B are sets such that there is a function $f : A \to B$ which is onto and $1-1$.

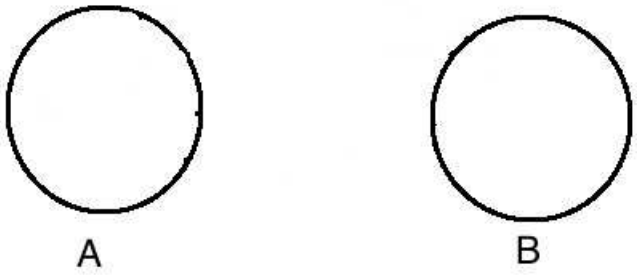
(1) Suppose that a is the number of elements in the set A and b is the number of elements in the set B. Suppose that $a < b$.
   (a) Let $f : A \to B$ be a function. Can f be onto? Make a picture.



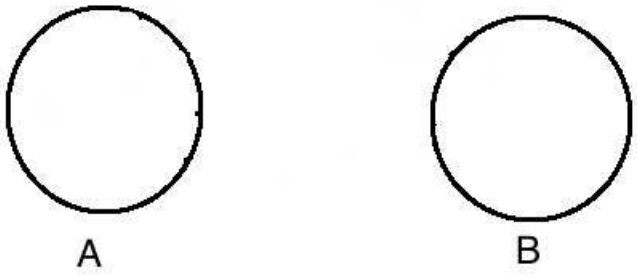
   (b) Can you always construct a function $f : A \to B$ which is $1-1$? How? Make a picture.



(2) Suppose that a is the number of elements in the set A and b is the number of elements in the set B. Suppose that $a > b$.
(a) Let $f : A \to B$ be a function. Can f be onto? Make a picture.



(b) Can you always construct a function $f : A \to B$ which is $1-1$? How? Make a picture.



(1) How do we compare infinite sets if we can not count the number of elements in them? What ideas can we use?

(2) Define a function from the set of odd numbers to the set of even numbers that is
- $1-1$
- onto

Make a conclusion.

(3) Define a function from the set of integers $\mathbb{Z} = \{...-3,-2,-1,0,1,2,3...\}$ to the set of natural numbers $\mathbb{N} = \{0,1,2,3,4...\}$ that is
- $1-1$
- onto

Make a conclusion.