

INTRODUCTION TO SETS

JUNIOR CIRCLE 05/22/2011

What is a set? In math, we use the word *set* to describe several objects taken together.

Here are some examples of sets you might be familiar with:

- a flock of birds;
- a school of fish;
- all students in the math circle;

(1) Let's discuss examples of naming sets:

(a) How do you call the set of cows grazing on a pasture together?

(b) How do you call the set of all soccer players who are training together and will play in a game?

(c) How do you call the set of all ships sailing together?

(2) Describe sets that can be called a

(a) Choir/Chorus

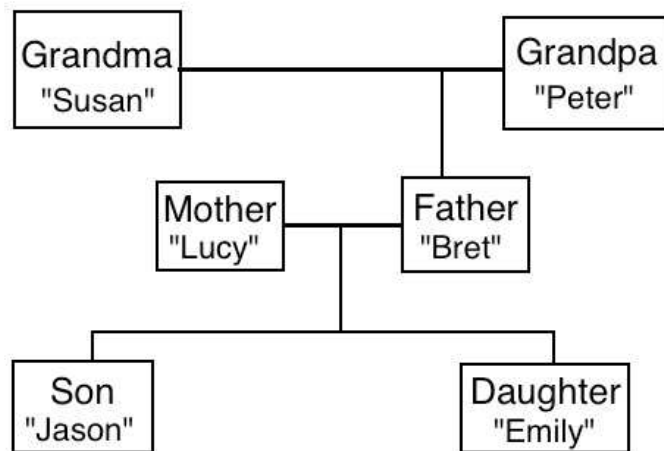
(b) Orchestra

(c) Collection

(d) Library

(e) Crew

(3) Here is a family tree:



(a) Circle the set of adults.

(b) Square the set of children.

(c) Put a triangle around the set of fathers.

(d) Does the set of fathers intersect with the set of sons?

(e) How can you call the set consisting of Susan, Lucy and Emily?

Subsets. Two sets are *equal* if they consist of the same elements.

- If a set contains no elements, it is called the empty set and is denoted by \emptyset .
 - for example, the set of 8 foot tall people is an empty set.
- Give your own example of a set which is empty:

If an element is in a set, we use “ \in ” to indicate that. If an element is not in a set, we use “ \notin ” to indicate that.

- For example, if $A = \{1, 2, 3\}$, then $2 \in A$ and $5 \notin A$.

A part of set is called a *subset*. For example, if $A = \{\text{set of all math circle students}\}$ and $B = \{\text{set of students at Rosalie's table}\}$ then B subset of A , which is denoted by $B \subset A$.

(1) Given the sets below, find which sets are subsets of another:

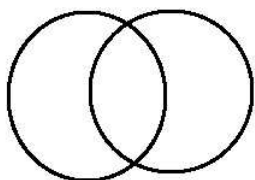
- $A =$ set of all flowers
- $B =$ set of all red objects
- $C =$ set of all tulips
- $D =$ set of all balloons
- $E =$ set of all things you can use for birthday decorations

(2) Give an example of two sets, A and B , such that:

- $A \subset B$
- $B \subset A$
- there are elements common to A and B .

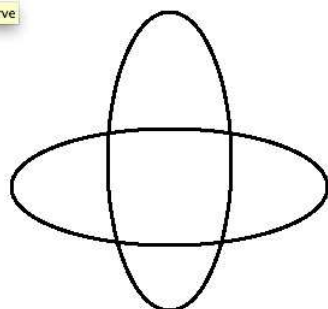
Intersection of Sets. The common elements sets form the *intersection* of sets: $A \cap B$.

(1) Shade the area representing the common part of the two given sets:

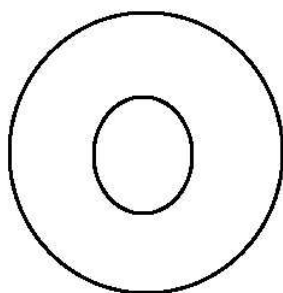


(a)

Curve

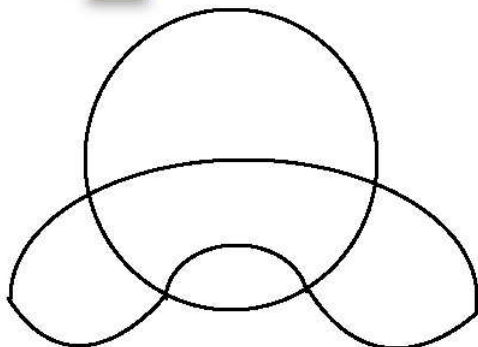


(b)



(c)

Selection



(d)

(2) Draw two triangles so that their intersection is:

(a) a hexagon

(b) a pentagon

(c) a quadrilateral

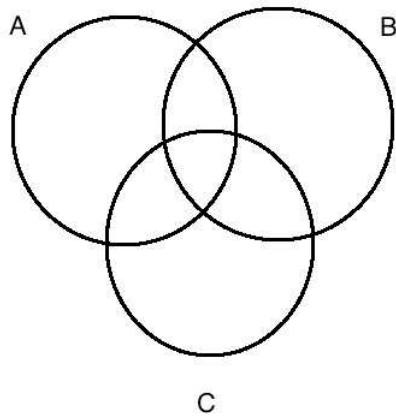
(d) a triangle

(e) a segment

(f) a point

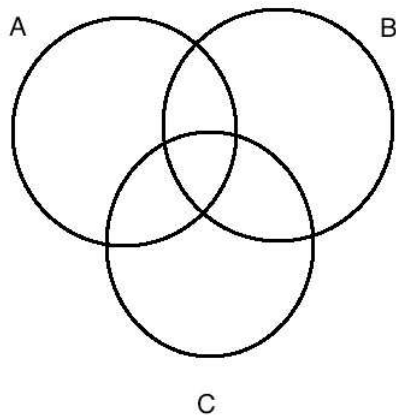
(g) the empty set

(3) Here is a diagram showing intersections between three sets:



(a)

- Shade $A \cap B$ yellow;
- Shade C green;
- Shade $(A \cap B) \cap C$ blue;



(b)

- Shade $B \cap C$ yellow;
- Shade A green;
- Shade $A \cap (B \cap C)$ blue;

(c) Compare $(A \cap B) \cap C$ and $A \cap (B \cap C)$ on the pictures above and make a conclusion;

(4) Put 4 elements (use dots to denote them) into the diagrams of sets A and B so that:

(a) A has 3 elements and B has 3 elements.

(b) A has 3 elements and B has 4 elements.

(c) A has 4 elements and B has 3 elements.

(d) A has 0 elements and B has 4 elements.

(e) A has 4 elements and B has 4 elements.

(f) A has 2 elements and B has 2 elements.

Union of Sets. The *union of two sets*, denoted by “ \cup ” (a symbol similar to U, the first letter in the word “union”), is the set of all elements that are in at least one of the sets. For example, if $A = \{1, 2, 5\}$ and $B = \{2, 6, 7\}$ then $A \cup B = \{1, 2, 5, 6, 7\}$

(1) Find how many elements are in $A \cup B$ if:

- There are 5 elements in A ;
- There are 7 elements in B ;
- There are 2 elements in $A \cap B$.

Hint: try drawing a picture.

(2) Suppose that $A \subset B$. Find $A \cap B$ and $A \cup B$ in terms of A and B .

Using sets to solve problems.

(1) In Ms. Kim's classroom:

- 25 students study French;
- 25 students study German;
- 18 students study both languages.

(a) How many students study at least one of the three languages?

(b) How many study French, but not German?

(c) How many study German, but not French?

(2) At a music camp, there were 12 pianists and 9 violinists. All together there were 16 people, How many people play both instruments?

- (3) How can you have four women so that there are
- 2 mothers;
 - 3 daughters;
 - 2 sisters;

among them?

- (4) A set A has x elements and a set B has y elements.
(a) suppose that $A \cap B = \emptyset$. Make a diagram.

(b) How many elements are there in $A \cup B$?

- (5) A set A has x elements and a set B has y elements.
(a) suppose that $A \cap B = z$. Make a diagram.

(b) How many elements are in $A \cup B$?

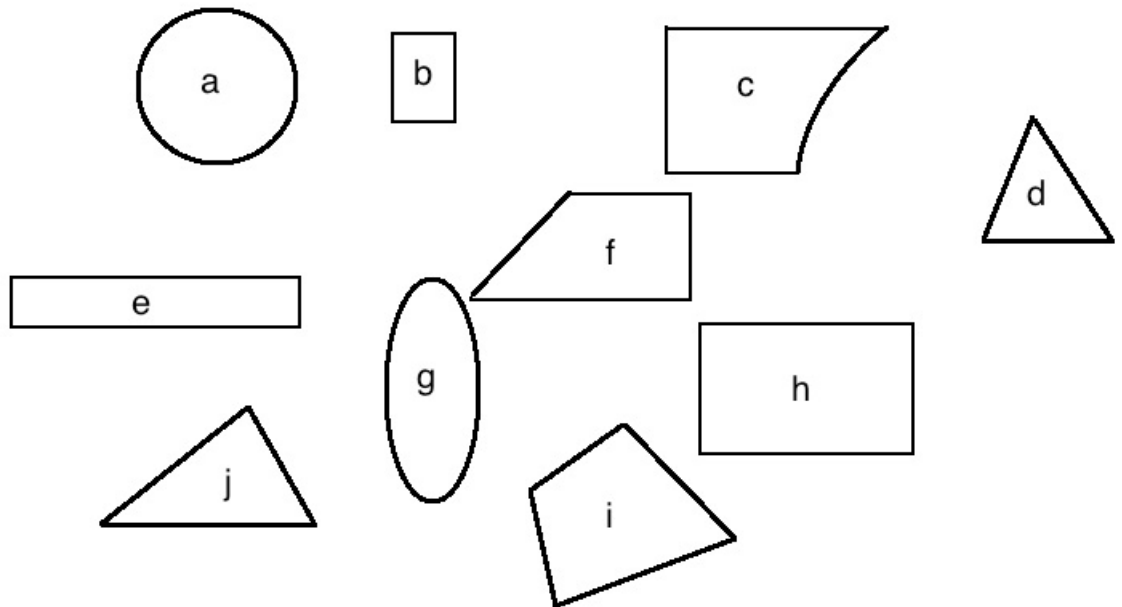
(6) A set A has x elements and a set B has y elements.

(a) Suppose that $A \cup B$ has u elements. Make a diagram.

(b) How many elements are there in $A \cap B$?

(7) Solve the following problem:

(a) Find all the right angles in the shapes below:



(b) Find all the shapes which are elements of the given sets:

(i) A is set of all polygons

$$A = \{ \quad \quad \quad \}.$$

(ii) B is the set of all triangles

$$B = \{ \quad \quad \quad \}.$$

(iii) C is the set of all quadrilaterals

$$C = \{ \quad \quad \quad \}.$$

(iv) D is the set of all rectangles

$$D = \{ \quad \quad \quad \}.$$

(v) E is the set of all squares

$$E = \{ \quad \quad \quad \}.$$

(c) Draw the Venn Diagram of the sets A, B, C, D, E . Write down all of the inclusions using \subset notation.