Homework 7: Young Tableaux

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1 Homework

Problem 1.
Let $d(n)$ be the number of partitions of n into distinct odd parts. Show that $p(n) - d(n)$ is even for all n.
Hint: think about reflecting Young tableaux across the diagonal and use problem L6.5.

Problem 2.
Let O, A, B be points on the plane. Let A', B' be points on the segments OA and OB respectively such that $OA' \cdot OA = OB' \cdot OB$. Show that $A'B'BA$ is a cyclic quadrilateral.

2 Reading

Solution 1 (H6.1).
How many ways are there to create an exam where every problem is worth at most 3 points and the whole exam is worth 100 points? Consider the problems unlabelled.

Proof. This problem is equivalent to counting the number of partitions of 100 into parts each of which is at most 3. Using the reflection of Young tableaux across the diagonal, this is the same as counting the number of ways to partition 100 into at most 3 parts. There is 1 way to split it into 1 part, and clearly 50 ways to split it into two parts – from 1,99 to 50,50. To split into 3 parts, first consider the number of ways to split 100 into 3 ordered parts. By stars and bars, it is equal to $\binom{99}{2} = 4851$. This number counts each partition into three distinct parts 6 times, and each partition where two parts are equal 3 times. Note that since 100 is not divisible by 3, there are not partitions where all three parts are equal. Let the number of partitions with distinct parts be x, and the number of partitions with two equal parts be y. Then we have $6x + 3y = 4851$, and we are looking for $x + y$. We can easily compute y – for every even number a between 1 and 99, there is exactly one partition $a/2, a/2, 100 - a$. Thus $y = 49$. Then

$$x + y = \frac{4851 + 3y}{6} = \frac{4998}{6} = 833$$

Adding the 51 partitions into at most 2 parts, we get the final answer 884. □