Lesson 2.5: Combinations and Pascal’s Triangle

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1 From Last Week

Problem 2.
Show that if \( n \geq 4 \) and \( 2 \leq k \leq n - 2 \), then
\[
\binom{n}{k} = \binom{n-2}{k-2} + 2 \binom{n-2}{k-1} + \binom{n-2}{k}
\]

Problem 4.
a) Show that if \( p \) is prime and \( 1 \leq k < p \), then
\[
p \mid \binom{p}{k}
\]
b) Show that if \( p \) is prime, then
\[
p \mid \binom{2p}{p} - 2
\]

Problem 5.
Let \( ABCD \) be a cyclic quadrilateral, and let \( T \) be the intersection of lines \( AB \) and \( CD \). Assume \( A \) lies on the segment \( TB \) and \( D \) lies on the segment \( TC \). Show that \( TA \cdot TB = TC \cdot TD \).

Problem 6.
Let \( BB_1 \) and \( CC_1 \) be altitudes in a triangle \( \triangle ABC \). Show that the tangent line at \( A \) to the circumcircle of \( \triangle ABC \) is parallel to \( B_1C_1 \).

2 New Problems

Problem 1.
Toys R Us has recently introduced a new revolutionary type of toy – a wire cube with a colored sphere at each corner. The spheres can be one of 8 colors, and each cube has to contains all 8 possible colors. How many different cubes can Toys R Us produce?

Problem 2.
Count the number of 5-digit numbers which contains exactly the digits 1,2,3,4,5 and the even digits are not adjacent to each other.