Lesson 2.5: Combinations and Pascal's Triangle

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1 From Last Week

Problem 2.
Show that if $n \geq 4$ and $2 \leq k \leq n-2$, then

$$\binom{n}{k} = \binom{n-2}{k-2} + 2\binom{n-2}{k-1} + \binom{n-2}{k}$$

Problem 4.
a) Show that if p is prime and $1 \leq k < p$, then

$$p \mid \binom{p}{k}$$

b) Show that if p is prime, then

$$p \mid \binom{2p}{p} - 2$$

Problem 5.
Let $ABCD$ be a cyclic quadrilateral, and let T be the intersection of lines AB and CD. Assume A lies on the segment TB and D lies on the segment TC. Show that $TA \cdot TB = TC \cdot TD$.

Problem 6.
Let BB_1 and CC_1 be altitudes in a triangle $\triangle ABC$. Show that the tangent line at A to the circumcircle of $\triangle ABC$ is parallel to B_1C_1.

2 New Problems

Problem 1.
Toys R Us has recently introduced a new revolutionary type of toy – a wire cube with a colored sphere at each corner. The spheres can be one of 8 colors, and each cube has to contains all 8 possible colors. How many different cubes can Toys R Us produce?

Problem 2.
Count the number of 5-digit numbers which contains exactly the digits 1,2,3,4,5 and the even digits are not adjacent to each other.