

# Lesson 1: Combinations

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For a positive integer  $n$  and  $0 \leq k \leq n$  let  $\binom{n}{k}$  denote the number of ways to choose  $k$  objects out of  $n$  distinct objects. For example,  $\binom{3}{1} = 3$  and  $\binom{4}{2} = 6$ .

**Problem 1.**

Show that

$$\binom{n}{k} = \binom{n}{n-k}$$

without using the formula in problem 2.

**Problem 2.**

Show that

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

**Problem 3.**

If  $n > 1$  and  $k > 0$ , show that

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

a) Algebraically, using the formula from problem 2.

b) Combinatorially, without using the formula.

**Problem 4.**

Consider an  $n \times m$  square grid. Show that the number of ways to travel along the grid lines from the bottom-left corner to the upper-right corner while always going up or right is equal to  $\binom{n+m}{n}$ .

**Problem 5.**

a) Show that

$$\binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n} = 2^n$$

b) Show that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots + (-1)^n \binom{n}{n} = 0$$

**Problem 6.**

Let  $\triangle ABC$  be a right triangle with  $\angle B = 90^\circ$ . If  $BH$  is the altitude,  $AH = a$  and  $HC = b$ , show that  $BH = \sqrt{ab}$ .