Homework 6: Quadratic equations V

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1 Reading

Solution 1 (L5.4).
Is it true that if $b > a + c > 0$, then the quadratic equation $ax^2 + bx + c = 0$ has two distinct real roots?

Solution:
Yes, it is true. To show that this equation has two distinct real roots, it is enough to show that it has a positive discriminant. Let us show that:

$$D = b^2 - 4ac > (a+c)^2 - 4ac = a^2 + 2ac + c^2 - 4ac = a^2 - 2ac + c^2 = (a-c)^2 \geq 0$$

This means that the discriminant is indeed positive, and we are done.

Solution 2 (L5.5).
All three coefficients of a quadratic equation are odd integers. Show that it cannot have a root of the form $1/n$, where n is an integer.

Solution: Suppose $1/n$ is a root of $ax^2 + bx + c = 0$ for a nonzero integer n. Then we can write

$$\frac{a}{n^2} + \frac{b}{n} + c = 0$$

$$a + bn + cn^2 = 0$$

Now let us look at the parity of n (quite a natural thing to do, since we are given the parity of a, b, c.) If n is even, then $bn + cn^2$ is even and a is odd, so $a + bn + cn^2 = 0$ is odd and thus nonzero. If n is odd, then each of a, bn, cn^2 is odd and so $a + bn + cn^2$ is odd again. Therefore it can never be 0, contradiction.

2 Homework

Problem 1.
For which values of a does the equation $\frac{a}{2}x^2 + (a+1)x + 1 = 0$ have two distinct real roots?

Problem 2.
Find all pairs of prime positive integers p, q such that the equation $x^2 + px + q = 0$ has two integer roots.