Problem 1.

a) For each of the following strings say whether the following DFA accepts it:

- 10011
- 10010010
- 11001010001110010110100000

b) Give an intuitive description as to what set of strings this DFA accepts.

Problem 2.

a) Write a DFA that accepts the set of binary numbers divisible by five.

b) Write a DFA that accepts the set of binary strings with the same numbers of zeroes as ones mod 3.
Problem 3.

a) Prove that if you run a DFA \( M \) with \( k \) states on a string \( \alpha \) of size \( n > k \) and it outputs yes, then there is a string \( \beta \) of size \( l \leq k \) such that \( M \) also outputs yes when you input \( \beta \).

b) Prove that if you run a DFA \( M \) with \( k \) states on a string \( \alpha \) of size \( n > k \) and it outputs yes, then \( \alpha \) can be written as \( xyz \) where \( y \) is not the empty string and \( M \) outputs yes on \( xy^mz \) for any \( m \), where \( y^m \) represents \( y \) written \( m \) times.

c) Show that there is no DFA which says yes on a string if and only if it is of the form \( 0^n1^n \) for some \( n \).

Problem 4.

Prove that if there is a DFA \( M \) reading left to right which accepts a certain set of strings, then there is also a DFA \( M' \) which reads right to left also accepting the same set of strings. (Note that this is the only time we allow DFA’s to scan left-to-right or right-to-left; always assume I mean a left-to-right scanning DFA in the above questions).