Lesson 6 Problem 2 Solution

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Problem 2.
Consider a connected component C of this graph. Every vertex in C has degree 2. Now let us pick a vertex v in C, and an edge from v to some w. Then let us move from v to w. Now we can start traversing C in the follows fashion: whenever we enter a new vertex u, we do it via some edge. Pick the second edge connected to u, and enter a new vertex using it. Since the number of vertices is finite, eventually we will come to an already visited vertex. But it cannot be anything other than v, since we never repeat edges, and returning to an already visited vertex which is not v would cause that vertex to have degree at least 3. Thus when we return to v, we have found a simple cycle. Notice that every vertex in the cycle has degree 2, so nothing else can be connected to it. Then this cycle is the whole of C, which means that every connected component is a simple cycle, which concludes the proof.