

Lesson 5 Problem 2

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Problem 2

a) Since being a bipartite graph forbids us to have edge inter white and black vertices, the maximal number of edges is achieved by connecting every black vertex with every white vertex. The total number of edges is then $b \cdot r$.

b) Suppose there are $n+x$ black vertices and $n-x$ white vertices. Connecting every white vertex with every black vertex, we have $(n+x)(n-x) = n^2 - x^2$ edges. Since $x^2 \geq 0$, this number is maximized when $x = 0$, the total number of edges is then n^2 .

c) Suppose there are $n+x+1$ vertices of one color and $n-x$ vertices for the other ($0 \leq x \leq n$). Connecting every white vertex with every black vertex, we have $(n+x+1)(n-x) = n^2 + n - x^2 - x$ edges. Since $x^2 + x \geq 0$, this number is maximized when $x = 0$, the total number of edges is then $n^2 + n$.