

Lesson 6: Bipartite Graphs and Geometry

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Problem 1.

Show that a graph is bipartite if and only if it does not have odd cycles.

Hint: parts and special cases of this problem have appeared in the last week's lesson and homework. Look through them to see what exactly you have left to prove.

Problem 2.

Show that a graph where every vertex has degree 2 is a collection of disjoint simple cycles.

Problem 3.

On a test every student solved exactly 2 problems, and every problem was solved by exactly 2 students.

a) Show that the number of students in the class and the number of problems on the test are the same.

b) The teacher wants to make every student present one problem they solved at the board. Show that it is possible to choose the problem each student presents so that every problem on the test gets presented exactly once.

Problem 4.

a) Let ABC be an acute triangle. Let AK and BL be altitudes, and call their intersection H . Show that $\angle BLK = \angle HCK$

b) Show that altitudes of an acute triangle intersect at one point.

Problem 5.

Let ABC be an acute triangle, and AA' , BB' and CC' be altitudes. Show that if $\angle BAC = 60^\circ$, then $\angle B'A'C' = 60^\circ$.