Problem 3.
We will show this by induction. Base: \(1/3 + 1/4 = 7/12 = 14/24 > 13/24\).
Step: by the induction hypothesis,
\[
\frac{1}{n+1} + \ldots + \frac{1}{2n} > \frac{13}{24}
\]
We want to show this when \(n\) is replaced by \(n+1\):
\[
\frac{1}{n+2} + \ldots + \frac{1}{2n+2} > \frac{13}{24}
\]
We will in fact show that the quantity on the left increased. Indeed, let us label
\[
\frac{1}{n+1} + \ldots + \frac{1}{2n} = S
\]
Then
\[
\frac{1}{n+2} + \ldots + \frac{1}{2n+2} = \left( \frac{1}{n+1} + \ldots + \frac{1}{2n} \right) + \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{n+1}
\]
\[
= S + \frac{1}{2n+1} + \frac{1}{2n+2} - \frac{1}{n+1}
\]
\[
= S + \frac{2n+2 + 2n+1 - 2(2n+1)}{(2n+1)(2n+2)}
\]
\[
= S + \frac{1}{(2n+1)(2n+2)} > S
\]
Therefore if \(S > 13/24\), then
\[
\frac{1}{n+2} + \ldots + \frac{1}{2n+2} > S > \frac{13}{24}
\]
which concludes the inductive step.