Lesson 4: Induction in Arithmetic

Konstantin Miagkov

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Problem 1.
Show that $n^5 - n$ is divisible by 5 for any positive integer n.

Problem 2.
a) Show that $n^2 + (n+1)^2 + (n+2)^2 + (n+3)^2 + (n+4)^2$ is divisible by 5 for any positive integer n.

b) Let m be a positive integer not divisible by 2 or 3. Show that $n^2 + (n+1)^2 + \ldots + (n+m-1)^2$ is divisible by m for any positive integer n. Hint: remember the formula

$$1^2 + 2^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

which was shown on the board during our very first induction class.

Problem 3.
Show that $3^{2n+2} + 8n - 9$ is divisible by 16 for any positive integer n.

Problem 4.
Kiselev 271, p. 101

Problem 5.
Show that in a quadrilateral $ABCD$ we have $\angle ABD = \angle ACD$ if and only if points A, B, C, D lie on the same circle. Such a quadrilateral is is called *cyclic* or *inscribed*.

Problem 6.
Show that if in a quadrilateral $ABCD$ we have $\angle ABC + \angle ADC = 180°$, then it is cyclic. This provides a converse to the problem 4 from last week, and gives us another characterization of a cyclic quadrilateral.