Problem 1.
Show that $n^5 - n$ is divisible by 5 for any positive integer $n$.

Problem 2.

a) Show that \( n^2 + (n+1)^2 + (n+2)^2 + (n+3)^2 + (n+4)^2 \) is divisible by 5 for any positive integer $n$.

b) Let $m$ be a positive integer not divisible by 2 or 3. Show that \( n^2 + (n + 1)^2 + \ldots + (n + m - 1)^2 \) is divisible by $m$ for any positive integer $n$. Hint: remember the formula

\[
1^2 + 2^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}
\]

which was shown on the board during our very first induction class.

Problem 3.
Show that $3^{2n+2} + 8n - 9$ is divisible by 16 for any positive integer $n$.

Problem 4.
Kiselev 271, p. 101

Problem 5.
Show that in a quadrilateral $ABCD$ we have $\angle ABD = \angle ACD$ if and only if points $A, B, C, D$ lie on the same circle. Such a quadrilateral is called cyclic or inscribed.

Problem 6.
Show that if in a quadrilateral $ABCD$ we have $\angle ABC + \angle ADC = 180^\circ$, then it is cyclic. This provides a converse to the problem 4 from last week, and gives us another characterization of a cyclic quadrilateral.