Weighing with Powers of 2

In the country of Binary Land, the factory makes only the following weights (in kilograms):

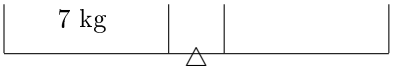
You can buy any number of these standard weights from the factory and use them with a balance scale.

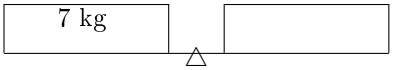
1. What is the pattern in the sequence of weights above?

2. Write down the next 3 weights in the sequence:

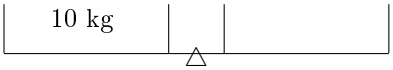
3. Balance each of the following objects using the weights 1, 2, 4, 8, 16, ...

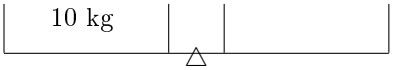
   (a) Find two different ways to balance a watermelon weighting 7 kilograms. (Note that you can use the same weight more than once).



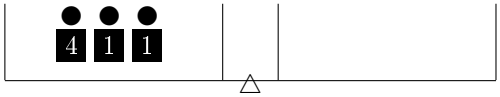


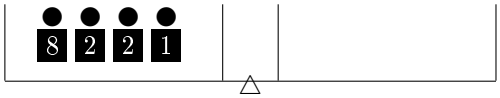
   (b) Find two different ways to balancs a metal ball weighing 10 kilograms. (Note that you can use the same weight more than once).

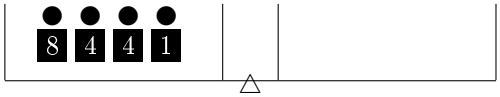


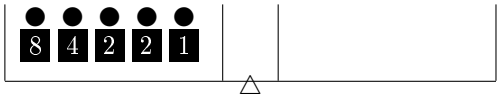


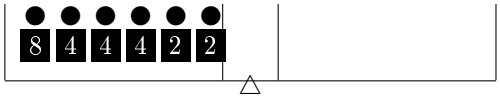
4. Now try to use each of the standard weight only once. Can you balance the following:

(a) 

(b) 

(c) 

(d) 

(e) 

5. Someone stole the 1 kilogram weight. With the remaining weights 

, ...).

(a) Can we balance 1 kg? Explain.

(b) 2kg?

(c) 3 kg?

(d) 4 kg?

(e) 10 kg?

(f) 15 kg?

(g) 20 kg?

(h) What can you say about the weights that you can balance when the 1 kg weight is missing?

6. The next day, the 2 kilogram weight got stolen, too. Which weights can you balance now? (You have the weights 

, ...).

(a) Can we balance 1 kg? Explain.

(b) 2kg?

(c) 3 kg?

(d) 4 kg?

(e) 10 kg?

(f) 15 kg?

(g) 20 kg?

(h) What can you say about the weights that you can balance?

7. They found the 1 kilogram weight! Which weights can you balance now? (You have the weights ).

(a) Can we balance 1 kg? Explain.

(b) 2kg?

(c) 4 kg?

(d) 8 kg?

(e) 10 kg?

(f) 12 kg?

(g) 20 kg?

(h) What can you say about the weights that you can balance?

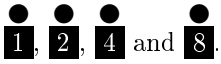
8. Balance the following. Do not use the same weight more than once. Fill in the table below:

   - if you are using a certain weight, put 1 in its column;
   - if you are not using a weight, put 0 in its column.

   Some examples are filled in:

Weight	Write as sum of weights	8	4	2	1
1	1 = 1	0	0	0	1
2	2 = 2	0	0	1	0
3	3 =				
4	4 =	0	1	0	0
5	5 =				
6	6 = 4 2	0	1	1	0
7	7 =				
8	8 =				
9	9 = 8 1	1	0	0	1
10	10 =				
11	11 =				
12	12 =				
13	13 = 8 4 1	1	1	0	1
14	14 =				
15	15 =				

9. Major robbery!

(a) After many weights were stolen you are left only with the weights 

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i. Can you balance 18 kilograms using each weight no more than once?

ii. Why or why not?

iii. What weights can you balance?

(b) What is the largest number you can write as a sum of some the numbers $1, 2, 4, 8$ without repeating any of the numbers?

(c) Balance 30 kilograms with the given weights

$$1, 2, 4, 8, 16, \ldots$$

without using any weight more than once. That is, write 30 as a sum of several of these numbers, without repeating any of the numbers:

$$30 = \square + \square + \square + \square.$$

Now write down the string of 0s and 1s that corresponds to 30. (Write 1 under the numbers that are used in the sum above. Write 0 under the numbers that are not used in the sum above):

	16	8	4	2	1
30					

We call this string of 0s and 1s the binary notation of the number 30. We will now circle the binary number every time we write it down so that we can tell the difference between binary notation and regular numbers.

(d) Can you write down 57 as sum of some of the numbers

$$1, 2, 4, 8, 16, 32, \ldots?$$

Do not use the same number twice.

$$57 = \square + \square + \square + \square.$$

Now write down the string of 0s and 1s that corresponds to 57:

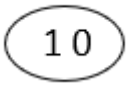
	32	16	8	4	2	1
57						

10. What number corresponds to the following strings of 0s and 1s. Fill in the table:

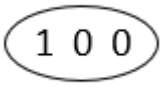
32	16	8	4	2	1	computation	number
0	1	1	0	1	0	$16+8+2=26$	26
0	1	1	0	1	0		
0	0	1	1	0	1		
1	1	0	0	1	0		
1	1	1	1	1	1		

11. Every number can be written in *binary notation*

(a) 1= 

(b) 2= 

(c) 3= 

(d) 4= 

Find the missing numbers or missing binary notation:

Binary -> Regular Numbers

(a) 

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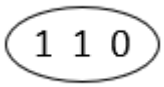
(b) 

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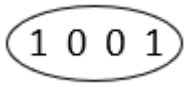
(c) 

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Regular Numbers -> Binary

(a) _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ = 6

(b) _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ = 7

(c) _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ = 8

12. Practice converting the following number into binary notation. Circle the binary notation

- 11 =

- 20 =

- 24=

- 15 =

- 35=

Practice converting the following number in binary notation to a regular number. Circle the binary notation.

- 1 0 1 0 =

- 1 1 1 1 =

- 1 1 1=

- 1 0 0 0 1 =

There are 10 kinds of people in the world. Those who know binary notaton and those who do not.

Explain the joke: