

Lesson 2: Induction in Combinatorics

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Problem 1.

Consider the following process. First, the board has the numbers 1, 1 on it. Then between every two numbers we write in their sum, to make 1, 2, 1. Now we do the operation for the second time, to make 1, 3, 2, 3, 1 and so on. Show that the sum of all numbers after 100 operations will be $3^{100} + 1$.

Problem 2.

Show that the $2^n \times 2^n$ board with *any* square cut out can be cut into angle-trominoes. *Hint: This looks harder than the problem we did on the board, but it is actually not!*

Problem 3.

$n \geq 3$ lines are drawn on a plane in such a way that no two lines are parallel and no three intersect at one point. Show that at least one of the pieces the lines cut the plane into is a triangle.

Problem 4.

Kiselev 268, p. 101.

Problem 5.

Let ABC be an equilateral triangle. Show that the circle with diameter AB goes through the midpoints of BC and CA .

Problem 6.

Let ABC be a right triangle with $\angle ABC = 90^\circ$. Suppose the circle with diameter AB intersects AC at the midpoint. Find the angles of ABC .