

# Lesson 1: Induction I

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**Problem 1.**

Show that the number  $111 \dots 111$  consisting of 243 ones is divisible by 243. Hint:  $243 = 3^5$ . This problem can be generalized as follows: for any positive integer  $n$ , the number consisting of  $3^n$  ones is divisible by  $3^n$ .

**Problem 2.**

a) Show that

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

b) Show that

$$1 + 3 + 5 + \dots + (2n-1) = n^2$$

**Problem 3.**

Show that  $n^3 + 2n$  is divisible by 3 for any positive integer  $n$ .

**Problem 4.**

a) Show that for any positive integer  $n$  we have  $2^n > n$ .

b) Find all positive integers  $n$  such that  $2^n > n^2$ .

**Problem 5.**

Suppose there are  $n$  lines drawn on a plane, in such a way that not two lines are parallel and no three lines intersect at the same point. Find a closed formula for the number of regions in which the lines split the plane.