

# Lesson 7: Games and Geometry II

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## **Problem 1.**

a) Two players are taking turns moving a *limp king* on a  $7 \times 7$  board. A limp king is a chess piece which can move one square to the right or diagonally to the right and up. The player who cannot make a turn loses. Who has a winning strategy?

b) Same problem, but now the limp king can also go up on square.

## **Problem 2.**

Two players are writing a sequence of digits on the board. A player loses if after their turn some number of consecutive digits on the board make a number divisible by 11. Who has a winning strategy?

## **Problem 3.**

Kiselev 239, page 91.

## **Problem 4.**

Kiselev 242, page 91.

## **Problem 5.**

Suppose the circle  $\omega$  is tangent to all the sides of the triangle  $ABC$  with  $AB = BC$ . Let  $K$  be the midpoint of  $AB$  and  $L$  be the midpoint of  $AC$ . Also let  $T$  be the intersection of  $\omega$  and  $KL$ . It is known that  $T$  does not lie on  $AC$ . Show that the tangent to  $\omega$  at  $T$  intersects the angle bisector of  $\angle ACB$  on the side  $AB$ .