

The Island of Knights and Liars

July 31st, 2016

Warm-up problems

1. The age of Peter's great grandfather is the smallest three digit number written with three *different* digits. How old is Peter's great grandfather?

First digit should be one (taking three digits to imply the leading digit is nonzero)

Next we can use 0 and 2 for the next to \rightarrow 102

1. Tara was prescribed a medication that she has to take every hour. She took the first pill at noon and has to take a total of 6 pills. When will she take the last one?

Took the first at noon (now there are 5) therefore 5pm.

1. It is raining at midnight on Tuesday. Do you think we can we expect sunny weather in 48 hours?

48 hours from exactly two days from now. Excluding serious astronomical anomalies, I wouldn't expect the sun to be out at midnight.

- 1.

Island of Knights and Liars

There are two types of inhabitants on the *Island of Knights and Liars*:

- *Knights* always tell the truth;
- *Liars* always lie;

From time to time, *Tourists* also visit the island. Tourists sometimes lie and sometimes tell the truth.

1. You arrive to the Island and meet two people, Tom and Tim. You ask each of them if he is a Knight.
 - Tom says that he is a Knight. Can you figure out who Tom is?
If Tom was a knight he would say “I am a Knight”, similarly a liar would say “I am a Knight.” Therefore we cannot draw any conclusions.
 - Tim says that he is a Liar. Can you figure out who Tim is?
We have discussed what both a liar and knight would say, i.e., “I am a Knight.” The only person that can then make the statement of “I am a Liar” is a tourist.
2. Sara and Tara live on the island. Sara says: “We are both Liars”. Can you determine who Sara and Tara actually are?
 - (a) Let's check this case by case:
 - i. Assume Sara is a Knight: Regardless of Tara's identity, Sara would not lie, implying that Sara is NOT a knight
 - ii. Assume Sara is a Liar: Since Sara is assumed to be a liar, she cannot tell the truth. The only way to resolve this is to have Tara be a knight.
 - A. Check this case: Is Sara then lying when she says they are both liars if Sara is a liar and Tara is a knight? Yes.

3. Mathew, Jack, and Kate are known to be Islanders (not Tourists!). You ask them if they are Knights.

- Mathew is shy, and mumbles something which you can't hear.
- You ask Jack what Mathew said. Jack says that Mathew said he (himself) was a Liar.
- Kate says "don't believe Jack, he is a Liar!"

Who are these Islanders? (*Note:* they are not tourists)

Once again, let us start by making some assumptions and checking their validity by looking for logical contradictions:

- Let's assume Kate is a knight and thereby telling the truth.
 - * Therefore we can take the "Don't believe Jack, he is a Liar!" as truth.
 - * This implies Jack is a Liar.
 - * If Jack is a liar then he MUST lie about what Matthew said, implying that Matthew said "I am a Knight".
 - So far we have Kate = Knight, Jack = Liar, and Matthew = Knight
- Let's assume Kate is a liar, and lying about Jack's statement.
 - * If this is the case then Jack is a Knight, and we can take Jack's statement as truth.
 - * Jack says that Matthew said he was a liar, but we know that NO ONE can make the statement of "I am a Liar" unless they are tourist, but in this case we have no tourist. Therefore, our assumption must be incorrect, and Kate must indeed be a Knight.
- This problem is easy to solve if you remember part 1; you learn that no islander ever makes the statement that they themselves are a Liar, which immediately lead us to the conclusion that Jack had to be a liar, skipping all my logical contradiction checking.
 - This is more difficult to recognize for younger students, and checking case by case is a good habit to have for the sake completeness early on.

4. While visiting the Island of Knights and Liars, I had a conversation with a local Knight. I asked him the same question twice, and he gave me two different answers. What could be my question?

- (a) Any question that asks about things that are nonconstant e.g.
- i. What time is it exactly?
 - ii. What am I wearing? (Before and after you change)

Fun problems

1. Kara added the age of her mom and the age of her dad and got 70. How soon (in how many years) will she get 80 when she adds up the age of her mom and the age of her dad?

$$M+D=70$$

$$(M+x)+(D+x)=80 \text{ implies } x=5$$

1. Andy took a number, added 2 to it, then subtracted 5, then doubled the number. He got 4. What number did Andy start with?

$$2*(x+2-5)=4$$

Backsolve to get 5.

1. One apple cost more than two bananas. What is more expensive: two apples or three bananas?

$$a > 2b$$

multiply both sides by $\frac{3}{2}$ and you will see that $\frac{3a}{2} > 3b$, which translates to 1 and a half apples cost more than 3 bananas, hence 2 apples MUST cost more than 3 bananas.