Fun and Games on a Chess Board

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I Names of squares on the chess board

Color the following squares on the chessboard below:

c3, c4, c5, c6, d5, e4, f3, f4, f5, f6

What letter do these squares form together? N
II How many squares are there on a chessboard?

A chessboard itself is a square with side 8.

1. The number of $1 \times 1$ squares on the chess board is $64$.

2. What about bigger squares?
   Let's first count squares of size $2 \times 2$:

   ![Diagram of 2x2 squares]

   Idea: Instead of counting $2 \times 2$ squares, we will count the small $1 \times 1$ squares which can serve as the left lower corners of the $2 \times 2$ squares that fit on the chessboard.

   First, shade the left lower corner of the $2 \times 2$ square above.
For each of the squares below, decide if it can be a left lower corner of a $2 \times 2$ square:

Remember, the $2 \times 2$ squares on the chessboard can overlap. Like this:

(a) square c3

(b) square g6

(c) square f8

(d) square h2
Now color all 1 × 1 squares that can serve as the left lower corners of a 2 × 2 square:

How many 2 × 2 squares can you fit onto a chessboard?

\[ 7 \times 7 = 49 \]
3. For each of the squares below, decide if it can be a left lower corner of a $3 \times 3$ square:

(a) square e6 \[ \text{Yes} \quad \text{No} \]

(b) square g3 \[ \text{Yes} \quad \text{No} \]

(c) square a7 \[ \text{Yes} \quad \text{No} \]

(d) square f6 \[ \text{Yes} \quad \text{No} \]

Now color \textit{all} $1 \times 1$ squares that can serve as the left lower corners of a $3 \times 3$ square:

How many $3 \times 3$ squares can you fit onto a chessboard?

\[ 6 \times 6 = 36 \]
Now you can fill out the table below:

<table>
<thead>
<tr>
<th>Type of Square</th>
<th>Number of such squares</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Square" /></td>
<td>$8 \times 8 = 64$</td>
</tr>
<tr>
<td><img src="image" alt="Square" /></td>
<td>$7 \times 7 = 49$</td>
</tr>
<tr>
<td><img src="image" alt="Square" /></td>
<td>$6 \times 6 = 36$</td>
</tr>
</tbody>
</table>

**Homework**

Count the number of $4 \times 4$, $5 \times 5$, $6 \times 6$ and $7 \times 7$ squares on the chessboard in the same way. In each case, use a chessboard picture to shade all the $1 \times 1$ squares that can be left lower corners of the bigger squares that fit completely onto the chessboard.
Now color all $1 \times 1$ squares that can serve as the left lower corners of a $4 \times 4$ square:

Color all $1 \times 1$ squares that can serve as the left lower corners of a $5 \times 5$ square:
Color all $1 \times 1$ squares that can serve as the left lower corners of a $6 \times 6$ square:

Color all $1 \times 1$ squares that can serve as the left lower corners of a $7 \times 7$ square:
Fill out the table below with the numbers of squares:

<table>
<thead>
<tr>
<th>size of the square</th>
<th># of squares of this size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 × 1</td>
<td>(8 \times 8 = 64)</td>
</tr>
<tr>
<td>2 × 2</td>
<td>(7 \times 7 = 49)</td>
</tr>
<tr>
<td>3 × 3</td>
<td>(6 \times 6 = 36)</td>
</tr>
<tr>
<td>4 × 4</td>
<td>(5 \times 5 = 25)</td>
</tr>
<tr>
<td>5 × 5</td>
<td>(4 \times 4 = 16)</td>
</tr>
<tr>
<td>6 × 6</td>
<td>(3 \times 3 = 9)</td>
</tr>
<tr>
<td>7 × 7</td>
<td>(2 \times 2 = 4)</td>
</tr>
<tr>
<td>8 × 8</td>
<td>(1 \times 1 = 1)</td>
</tr>
</tbody>
</table>

Now add up all the numbers in the right column to find the total number of squares of all sizes.

\[1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = 204\]