

# Homework 8: Euclid's lemma

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**Problem 1.**

Let  $n$  be a positive integer. Can  $n - 2017$ ,  $n$  and  $n + 2017$  all be prime numbers?

**Problem 2.**

Show that the product of 3 consecutive integers is divisible by 6.

**Problem 3.**

Let  $[x, y]$  denote the *least common multiple*, or LCM of  $x$  and  $y$  – the smallest positive integer that is divisible by  $x$  and  $y$ . Also here  $(x, y)$  denotes the GCD of  $x$  and  $y$ . Solve the following systems of equations in the positive integers:

a)

$$\begin{cases} (x, y) = 5 \\ [x, y] = 10 \end{cases}$$

b)

$$\begin{cases} (x, y) = 1 \\ [x, y] = 4 \end{cases}$$

c)

$$\begin{cases} (x, y) = 5 \\ [x, y] = 31 \end{cases}$$