

Homework 6 Problem 3 Solution

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Problem 3.

Answer: 1. Let $d = \gcd(n, 36)$. By contradiction, suppose it was bigger than 1. It certainly must be a divisor of 36, so it is one of 2, 3, 4, 6, 9, 12, 18, 36. Note that each of the numbers on this list is divisible by 2 or 3. So $2 \mid d$ or $3 \mid d$. But d is a divisor of n , so $2 \mid n$ or $3 \mid n$. Suppose $2 \mid n$. Then $2 \mid n^2$, and consequently $n^2 + 1$ has remainder 1 when divided by 2. But by the problem statement $n^2 + 1$ is divisible by 6, so it cannot have remainder 1 when divided by 2. Similar argument works for 3: if $3 \mid n$, then $3 \mid n^2$, and so $n^2 + 1$ is not divisible by 3, contradicting it being divisible by 6.