

# Lesson 7: Remainders, Primes and the Euclidean Algorithm

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**Problem 1.**

Compute: the remainder of  $-7$  when divided by  $-2$ , the remainder of  $-153$  when divided by  $15$ , the remainder of  $153$  when divided by  $-15$ .

**Problem 2.**

Show that a prime number greater than  $3$  can be expressed as  $6n+1$  or  $6n+5$  for some nonnegative integer  $n$ .

**Problem 3.**

- a) Find 3 distinct positive integers greater than  $1$  such that product of any two is divisible by the third.
- b) Show how to construct infinitely many such examples.

**Problem 4.**

- a) Let  $a, b$  be positive integers such that  $a \leq 100$  and  $b \leq 100$ . Show that computing  $\gcd(a, b)$  with the Euclidean algorithm takes at most  $20$  steps.
- b) Show that in fact it takes at most  $11$  steps.