Lesson 5 Problem 4 Solution

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a) Note that if 5^a and 2^a both divide some number n, then so does $2^a \cdot 5^a = 10^a$ because 2 and 5 are prime and thus there must be at least a 2's and 5's in n's prime factorization. Also, if n ends with a zeroes, then $10^a \mid n$ and thus 2^a and 5^a both divide n. So we just need to find the largest a such that 2^a and 5^a both divide $n!$.

Again, since 2 and 5 are both prime, we just need to calculate the number of times they occur in the prime factorization of $n!$. Since $10! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 = 2 \cdot 3 \cdot 2^2 \cdot 5 \cdot 2 \cdot 3 \cdot 7 \cdot 2^3 \cdot 3^2 \cdot 2 \cdot 5 = 2^8 \cdot 3^4 \cdot 5^2 \cdot 7$, two is the smallest number such that both 2^2 and 5^2 divide $n!$, so $n!$ ends in two zeroes.

b) We can proceed like in part a), except instead of prime factorizing every number between one and a hundred (although this is a possible solution), we can just count the number of multiples of 5 in the range of 1 to 100 inclusive, and then add it to the number of those multiples which are multiplied by another 5, and add it to the number of those multiples that are multiplied by another 5 and so on. Except since $5^3 = 125 > 100$, we just need to check the first two cases (numbers divisible by 5 and numbers divisible by 25).

Note that we don't need to find the largest power of 2 which divides 100! since two appears as a prime factor of numbers in the interval [1,100] more often than 5. Finally, since 5^1 goes into 100 twenty times, and 5^2 goes into 100 four times, the highest power of 5 such that 5^a divides 100! must be $20 + 4 = 24$.