Problem 2

a) A positive integer $n$ that has remainder 7 when divided by 9 will have $n = 9m + 7$ where $m$ is a nonnegative integer. Therefore when being divided by 3, $n = 9m + 7$ will have the same remainder as $7/3$ since $9m$ is divisible by 3. $7/3$ has remainder 1, so clearly $n$ cannot have remainder 2 when being divided by 3.

b) Suppose it can, $n = 144m + 23 = 90k + 29$ where $k$ and $m$ are nonnegative integers. Consider the remainder of $n/9$. When being divided by 9, $n = 144m + 23$ will have the same remainder as $23/9$ since $144m$ is divisible by 9 (This can be checked with the divisibility criterion of 9: $1 + 4 + 4 = 9$). $23$ has remainder 5 when divided by 9. With similar argument, $n = 90k + 29$ will have the same remainder as 29 when divided by 9. 29 has remainder 2 when divided by 9, which is a contradiction.