1. What is work?

Work is a word we use to describe mental or physical activity. It’s a very vague word, used to describe many different kinds of activities. Importantly, work doesn’t have to be boring or tedious. Fun things like eating ice cream or swimming laps in the pool are also considered work. Work can even be done by inanimate objects! If a pipe is filling a pool with water, that is a type of work being done by the pipe. Answer the following questions about work.

1) Sarah and Rose are pulling weeds from their garden. At the same time, Mike is pulling weeds from his garden. Once all of the weeds have been pulled, all three have each done the same amount of work (pulled the same number of weeds). Suppose Sarah and Rose pulled 20 weeds together. How many more weeds did Sarah and Rose’s garden originally have than Mike’s garden?
(2) Two pipes, a narrow pipe and a wide pipe, lead into a swimming pool. In one minute, the narrow pipe can deliver 2 gallons of water and the wide pipe can deliver 5 gallons of water.

(a) What is the “work” being done in this case, and who/what is performing the work?

(b) In one minute, how much more water is delivered by the wide pipe than the narrow pipe?

(c) In two minutes, how much more water is delivered by the wide pipe than the narrow pipe?
(3) Anna and Daria are working together to fill water balloons. After one hour, they have successfully filled 60 water balloons.

(a) Suppose Anna did three times much work as Daria. How many water balloons did Anna fill?

(b) What percentage of the work was done by Anna?

(c) What percentage of the work was done by Daria?
2. What is rate of work?

As you saw in the previous problems, when we discuss work we normally also discuss time. It’s helpful to know how much work can be done. It’s more helpful to know how much time it will take to complete that work. For this reason, we would like a way to describe how quickly work can be done. For all problems in this packet, we will assume that work is done at a constant rate. This means the work does not slow down or speed up. Although this does not always happen in real life, it is still useful for estimating the average rate of work. For example, a car does not travel at constant speed. But, it is still useful to calculate the average speed at which the car travels.

(1) Aaron is biking to UCLA. In ten minutes, Aaron bikes 2 miles.
   (a) If he continues biking at the same rate, how many miles can Aaron bike in one hour? In other words, how many “miles per hour” is Aaron traveling?

   (b) How many miles can Aaron bike in two hours?

(2) Suppose Rohan can eat 120 Skittles in three hours at a constant rate.
   (a) How many “Skittles per hour” can Rohan eat?

   (b) If Rohan eats Skittles at this rate for a week straight, how many Skittles will he eat?
3. Working Together

Use your new knowledge about work and rate of work to solve the following problems. When multiple people work together, assume they are working on the problem simultaneously (at the same time), not one after the other.

(1) Five rabbits eat five carrots in five minutes. How many rabbits would it take to eat ten carrots in ten minutes?

(2) Six boys wash six windows in 30 minutes. How many boys would it take to wash 48 windows in 3 hours?

(3) John and Jane can clean the house in 4 hours. Jane works four times as fast as John.
   (a) What percentage of the work was done by John?

   (b) What percentage of the work was done by Jane?
(c) How long would it take for John to clean the house by himself?

(d) How long would it take for Jane to clean the house by herself?

(4) John can mow the lawn in 60 minutes. His brother Peter can do it in 30 minutes. How fast can they do it if they work together?

(5) Three teachers were grading an exam. If they would work by themselves, it would take them 8 hours to do all the work. However, 2 hours after they started, several colleagues joined them. In the end, all work was done in just 4 hours (since the moment the first three teachers started). How many colleagues joined the teachers?
(6) The wide pipe fills the pool in 3 hours. The narrow fills the pool in 9 hours. How long would it take for the two pipes to fill the pool if both are open at the same time?

(7) The first (narrow) pipe fills the pool in 15 hours. The second (wide) pipe fills the pool in 10 hours. How long does it take for the two pipes to fill the pool if both are used at the same time?
   (a) What portion of the pool does the first pipe fill in 1 hr?

   (b) What portion of the pool does the second pipe fill in 1 hr?

   (c) What portion of the pool do the two pipes fill in 1 hr?

   (d) How many hours does it take for the two pipes to fill the pool?

   (e) What percentage of the pool was filled by the narrow pipe?
(f) What percentage of the pool was filled by the wide pipe?

(8) The first crew can repair a road in 9 days. The second crew can do the same work in 12 days. The first crew worked on the road for 3 days. After that, the second crew took over and finished the work. How long did the second crew have to work to finish the job?

(9) There are two pipes, one supplies hot water and the other supplies cold water. The pipe with hot water can fill a tank in 23 minutes, while the pipe with cold water can fill the same tank in only 17 minutes. If you open the hot pipe first, how soon do you need to open the cold pipe so that by the end you have one and a half times more hot water than cold water?
(10) If two people work together, it takes them 12 days to complete the job. If the first person does half the work and the second person does the other half, it takes them 25 days to complete their task. How long would it take for each?

(11) Thirty workers can complete a job in 12 days working 8 hours per day. If the job is to be completed in 10 days by twenty-four men, how many hours must they work per day?

(12) It takes a horse two days to eat a pile of hay. It takes a cow three days to eat a pile of hay, and it takes a lamb six days to eat a pile of hay. How long would it take them if they ate the pile of hay together. What percentage of the hay would the horse eat? The cow? The lamb?
(13) A pipe fills a pool in three hours. The drain empties it in five hours. How long would it take to fill the pool if you are filling it with the first pipe while the drain is left open?

(14) There are two ropes. It takes each rope 1 hour to burn completely. However, the burning is not uniform (that is, a rope can first burn fast and then slow down). How can you measure 45 minutes using these two ropes? (Hint: Think what you can do with the ropes).

(15) (An ancient problem) It takes a man 14 days to drink a barrel of water. If both he and his wife drink the water, it takes them 10 days to finish the barrel. How long would it take for the wife to finish the barrel by herself?
(16) (Math Kangaroo Problem) A complete set of dominoes contains 28 pieces. The pieces show every possible combination of two numbers of dots from 0 to 6 inclusive. How many dots are there altogether in a complete set of dominoes?

(17) (Math Kangaroo Problem) Adam and Tom are walking in the same direction around a circular table and counting chairs. They begin their counts with different chairs. Tom’s twentieth chair is Adam’s fourth chair, while Tom’s tenth chair is Adam’s forty sixth chair. How many chairs are there at the table?