Lesson 4 Problem 5 Solution.

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Problem 5.
Suppose none of them are divisible by 3. Suppose x has remainder 1 when it is being divided by 3, then $x = 3n+1$ and $x^2 = 9n^2+6n+1 = 3(3n^2+2n)+1$. So x^2 has remainder 1 when being divided by 3. Suppose x has remainder 2 when it is being divided by 3, then $x = 3n + 2$ and $x^2 = 9n^2 + 12n + 4 = 3(9n^2+12n+1)+1$. So x^2 has remainder 1 when being divided by 3. Similar argument goes for y and z. We can conclude that all of x^2, y^2 and z^2 have remainder 1 when being divided by 3. Suppose $x^2 = 3k+1$ and $y^2 = 3m+1$, then $x^2 + y^2 = 3(k + m) + 2$, having remainder 2 when divided by 3. This is a contradiction to $z^2 = x^2 + y^2$ having remainder 1 when divided by 3. So there must be at least one number divisible by 3 among x, y and z.