Lesson 4 Problem 1 Solution

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1 Problem 1

a) Notice how the linear graph seems to "repeat" every 15 squares. We can make this notion more rigorous by seeing that if (x_0, y_0) is an integer point, then $7 \cdot (x_0 + 15)/15 + 1/3 = 7 \cdot x_0 + 1/3 + 7 \cdot 15/15 = y_0 + 7$ is also an integer so $(x_0 + 15, y_0 + 7)$ is an integer point as well. Thus, another integer point of the graph is $(25, 12)$.

b) We can use the same logic as in part a) here, except now the number of squares for the line to repeat will be the denominator of the slope like in part a). We can ensure that the slope is a rational number since if a line passes through two different integral points (x_0, y_0) and (x_1, y_1) then we can calculate the slope to be $\frac{y_1 - y_0}{x_1 - x_0}$ which allows us to see that the denominator divides $x_1 - x_0$ (it may not be equal to this due to cancellation but that won't end up mattering). So if we plug in $x_1 + x_1 - x_0$ for x in $y = \frac{y_1 - y_0}{x_1 - x_0} \cdot x + b$ we get $\frac{y_1 - y_0}{x_1 - x_0} \cdot (x_1 + x_1 - x_0) + b = \frac{y_1 - y_0}{x_1 - x_0} \cdot x_1 + b + y_1 - y_0 = y_1 + y_1 - y_0$, which is certainly an integer since it is the sum and difference of integers. Thus, $(x_1 + x_1 - x_0, y_1 + y_1 - y_0)$ is another integer point.

c) One may be tempted on using the methods of parts a) and b) to create a new integer point by going a distance far enough to be divisible by the denominator of the slope, but this relies on the slope being rational. In fact, if we have a line like $y = \sqrt{2} \cdot x$, then it will never "repeat" like the past ones by hitting a new integer point because $n\sqrt{2}$ is only an integer when $n = 0$ (or else $\sqrt{2}$ would be rational).