

# Homework 4: Algebra and remainders

Konstantin Miagkov

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**Problem 1.**

a) Show that a number is divisible by 3 if and only if the sum of its digits is divisible by 3.

b) Show that a number is divisible by 9 if and only if the sum of its digits is divisible by 9.

c) Show that a number is divisible by 11 if and only if the alternating sum of its digits is divisible by 11. By alternating sum we mean the last digit minus the next to last digits plus the next one and so on. For example, the alternating sum of digits of 712465 is  $5 - 6 + 4 - 2 + 1 - 7 = -5$ .

**Problem 2.**

Let  $p, q$  be prime numbers greater than 3. Prove that  $p^2 - q^2$  is divisible by 24.

**Problem 3.**

For which positive integers  $n$  is

$$\frac{n^2 + 1}{n + 1}$$

an integer?