Lesson 4: Algebra and remainders.

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Problem 1.

a) The straight line \( y = \frac{7x}{15} + \frac{1}{3} \) passes through two integral points: (10, 5) and (-20, -9). Does it pass through any other integral points?

b) The graph of a function \( y = kx + b \) passes through two distinct integral points. Are there any other integral points on this graph?

c) Does there exist a linear function \( y = kx + b \) such that its graph passes through exactly one integral point?

Problem 2.

Solve the equation:

\[
\begin{cases}
\frac{x}{x+1} + y^2 = 4 \\
y^2 - \frac{5x}{x+1} = -14
\end{cases}
\]

Problem 3.

a) Let \( a, b \) be positive integers. Show that their exist unique nonnegative integers \( q, r \) such that \( a = bq + r \) and \( r < q \).

b) Let \( a, b \) be integers. Show that their exist unique integers \( q, r \) such that \( a = bq + r \) and \( 0 \leq r < |q| \).

Problem 4.

Show that \( n^5 + 4n \) is divisible by 5 for any integer \( n \).

Problem 5.

Let \( x, y, z \) be integers such that \( x^2 + y^2 = z^2 \). Show that at least one of \( x, y, z \) is divisible by 3.

Problem 6.

Is it possible to write 1986 as a sum of 6 squares of odd numbers?