Lesson 2: Tilings and colorings

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Throughout this whole handout, the tiling pieces can be rotated and reflected in any way.

**Problem 1.**
An 8 \times 8 chess board has the bottom left and top right corners cut out. Can it be tiled by 1 \times 2 dominoes?

**Problem 2.**
\begin{enumerate}
\item An 8 \times 8 chess board has the squares C6 and G2 cut out. Can it be tiled by 1 \times 2 dominoes?
\item Same question, except with C5 and G2 cut out.
\end{enumerate}

**Problem 3.**
Is it possible to tile a 10 \times 10 square with T-tetrominoes?

A T-tetromino looks like this:

\begin{center}
\begin{tikzpicture}
\draw (0,0) -- (1,0) -- (1,1) -- (0,1) -- (0,0);
\end{tikzpicture}
\end{center}

**Problem 4.**
A piece of cheese has a shape of a 3 \times 3 \times 3 cube with the central cube removed. A mouse starts eating a corner cube, and after finishing a cube moves to one of the adjacent cubes. Can the mouse eat all the cheese?

**Problem 5.**
Show that the 10 \times 10 board cannot be tiled with 1 \times 4 rectangles.

**Problem 6.**
Is it possible to tile an 8 \times 8 board with a corner cut out with 1 \times 3 rectangles?

**Problem 7.**
Is it possible to tile an 8 \times 8 board with 32 1 \times 2 dominoes in such a way that 17 of them are horizontal and 15 are vertical?