Cryptarithms*

SOLUTIONS!

April 30th, 2017

Warm-up problems

1. Andy and Ben live on the Island of Knights and Liars. Andy says: “We are both Liars”. Who is Andy and who is Ben? (Note: since they live on the island, they are not tourists).
   \[
   \text{Andy} : \text{Liar} \\
   \text{Ben} : \text{Knight}
   \]

2. Alice and Bob live on the Island of Knights and Liars.
   - Alice says: “We are the same kind”;
   - Bob says: “We are different kinds”.
   \[
   \text{Alice} : \text{Liar} \\
   \text{Bob} : \text{Knight}
   \]

   Who is Alice and who is Bob?

3. Insert parentheses (if necessary!) to make correct equalities:
   \[
   \left(12 + 6\right) \div 3 + 3 = 9
   \]
   \[
   12 + 6 \div \left(3 + 3\right) = 13
   \]
   \[
   12 + 6 \div 3 + 3 = 17
   \]

*The problems on cryptarithms are taken from the book "Sideways Arithmetic from Westside School" by Louis Schar.

Cryptarithmetic are mathematical puzzles in which the digits are replaced by letters of the alphabet. Remember that there are 10 digits: 0, 1, . . . , 8, 9. Moreover, each letter represents the same digit throughout the problem.

To solve a cryptarithm means to find what digits correspond to what letters so that you get a valid mathematical equality.

Solve the following cryptarithmetic. (Note that sometimes several solutions are possible).

1. \[
\begin{array}{c}
E G G \\
+ E G G \\
\hline
P A G E
\end{array}
\]

\[
\begin{array}{c}
8 8 \times 8 \\
\hline
1 7 9 8
\end{array}
\]

\[
E = 8 \quad G = 9 \quad A = 7 \quad P = 1
\]

2. \[
\begin{array}{c}
S H E \\
+ E E L \\
\hline
E L S E
\end{array}
\]

\[
\begin{array}{c}
2 \times 8 \\
\hline
1 0 9 1
\end{array}
\]

\[
S = 9 \quad H = 8 \quad E = 1 \quad L = 0
\]
3. \[ \frac{M O M}{Y O Y O} = \frac{1}{0506} \]

\[ M = 2 \quad O = 5 \quad Y = 0 \quad p = 3 \]

4. \[ \frac{S T A Y S}{T R U S T} = \frac{89718}{90589} \]

\[ S = 8 \quad T = 9 \quad A = 7 \quad R = 0 \quad Y = 1 \quad U = 5 \]
5. \[ \begin{array}{c}
\text{F O U R} \\
\text{S E V E N}
\end{array} \]
\[ \begin{array}{c}
\text{E L E V E N}
\end{array} \]
\[ + \begin{array}{c}
\text{O O O O O}
\end{array} \]
\[ + \begin{array}{c}
\text{S O 2 0 3}
\end{array} \]
\[ \begin{array}{c}
\text{0 1 0 2 0 3}
\end{array} \]

\[
O = 0 \quad U = 0 \quad F = 0 \quad R = 0 \quad S = 1 \quad E = 0 \quad V = 2 \quad N = 3
\]
\[
L = 1
\]

6. \[ \begin{array}{c}
\text{Y O Y O}
\end{array} \]
\[ \begin{array}{c}
\text{P O P}
\end{array} \]
\[ \begin{array}{c}
\text{P O P}
\end{array} \]
\[ \begin{array}{c}
\text{P O P}
\end{array} \]
\[ \begin{array}{c}
\text{Y O T O}
\end{array} \]
\[ \begin{array}{c}
\text{+ P O P}
\end{array} \]
\[ \begin{array}{c}
\text{+ P O P}
\end{array} \]
\[ \begin{array}{c}
\text{\Rightarrow 4 5 0 5}
\end{array} \]
\[ \begin{array}{c}
\text{1 0 1 0}
\end{array} \]

\[
P = 5 \quad Y = 1 \quad O = 0
\]
Homework

Adam and Beth live in an apartment building. They want to start at the top floor, go down, and come back to the top floor. Adam is taking the elevator. Beth is taking the stairs.

- When Beth runs down the stairs, she is twice as fast as the elevator;
- When Beth runs up the stairs, she is twice as slow as the elevator.
- The speed of the elevator is the same independently of whether the elevator goes up or down.

Who will come back to the top floor first? Make pictures to explain your solution.

Pictures

Adam first

Hint: Think of this problem in stages: Start by thinking who will reach the bottom floor first? Where will the other person be at this moment?