Math Fest.

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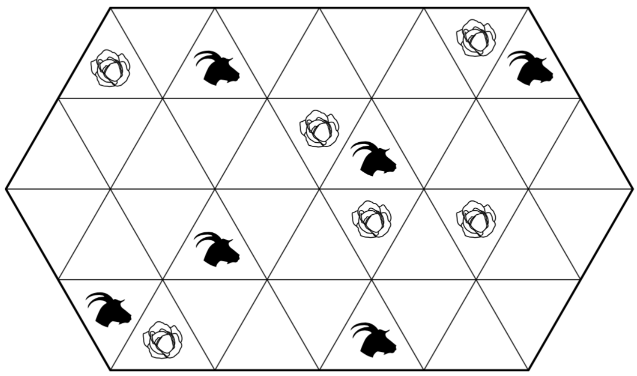
Problem 1 (13 point total)**.**
A farmer has divided his estate into triangular regions, each with side 50 meters. In some regions he planted cabbage, in some he keeps the goats. Help the farmer separate the cabbage from the goats by building additional fences along the sides of the triangles inside the estate.



a) [1 point] Make the total length of inner fences at most 700 meters.

b) [4 points] Make the total length of inner fences less than 700 meters.

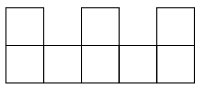
c) [8 points] Make the total length as small as possible and prove why it cannot be smaller.

Problem 2 (4 points)**.**
The pharmacist has 3 weights, using which he measured out 100 grams of hydrogen peroxide for one customer, 101 grams of honey for the other and 102 grams of iodine for the last. The pharmacist always puts weights on one side of the scale, and the product on the other. Could all 3 weights be less than 90 grams? If so, give an example, if not, explain why.

Problem 3 (6 points)**.**
Let $ABCD$ be a square. A point K is marked on the line AC beyond the point C such that $BK = AC$. Find $\angle BKC$ with proof.

Problem 4 (8 points total)**.**
Is it possible to place all integers from 1 to 8 in the squares of the figures below so that if the figure is cut into two pieces along the grid lines in any way, the sum of numbers in one piece always divides the sum in the other? For both parts, each integer has to be used exactly once.

For both shapes a) and b) either provide an example, or explain why none exist. The parts of the problem are independent, so you can solve one of them without solving the other and get points.
a) [3 points]



b) [5 points]