

Basic Probability!

January 18, 2015

In this handout we will continue to learn about probability.

1. We have a standard coin with one side that we call “heads” (H) and one side that we call “tails” (T).
 - (a) Let’s say that we flip this coin 100 times.
 - i. How many times do you expect to get “heads” as an outcome?

 - ii. In an actual experiment, can one get a different number of “heads” when flipping a coin 100 times?

 - iii. Do you think it is likely to get only 10 “heads”?

- iv. Do you think it is likely to get at least 55 “heads”?
- (b) Now we flip this coin 1000 times. How many times do you expect to get “heads”?
- (c) How is the number of “heads” you expect to get related to the number of flips?
- (d) We say that the probability of getting “heads” is $1/2$. A probability is:

$$\frac{\textit{number of ways to get something}}{\textit{total number of possibilities}}$$

(This formula is true when all the possible outcomes are equally likely. For example, when flipping a fair coin, the two possible outcomes “heads” and “tails” are equally likely.)

- i. What does the “1” in the numerator mean?

ii. What does the “2” in the denominator mean?

iii. What is the probability of “tails” if you flip a coin once?

2. Now let’s flip two coins at the same time.

(a) What are all the possible outcomes? (Denote “heads” by H and “tails” by T when listing outcomes.)

(b) What is the probability of getting two “heads”?

(c) If you flip these 2 coins 400 times, how many times do you expect to get two “heads”?

(d) How is your answer in part (c) related to the number of flips and the probability of getting two “heads”?

3. Now we flip a coin 3 times in a row.

(a) What are all the possible outcomes?

(b) What is the probability of getting exactly 3 “heads”?

(c) What is the probability of getting exactly 2 “heads”?

(d) What is the probability of getting exactly 1 “head”?

(e) What is the probability of getting 0 “heads”?

(f) What is the probability of getting at least two “heads”? (*At least two* means two or more.)

(g) What is the probability of getting at most 1 “head”? (*At most one* means one or less.)

(h) What is the relationship between your answers in parts (f) and (g)?

4. We roll a single die.

(a) What are the possible outcomes of our roll?

(b) What is the probability of rolling a 3?

(c) What is the probability of rolling an even number?

(d) What is the probability of rolling an odd number?

(e) What is the relationship between your answers in parts (c) and (d)?

(f) What is the probability of rolling at most a 4?

(g) What is the probability of rolling at least a 5?

(h) What is the relationship between your answers in parts (f) and (g)

(i) Based on your answers in parts (e) and (h) above, what can you say about the sum of probabilities of all outcomes of an experiment?

5. We color the faces of a single die: 4 faces are red and 2 faces are blue.

(a) Olga says that since we colored the die with two colors, it follows that the probability of rolling a red face is $1/2$ and the probability of rolling a blue face is $1/2$. Is she correct?

(b) What are the correct probabilities for rolling a red

face and rolling a blue face?

6. Create your own probability problem and solve it here!

7. Now we roll 2 dice, one after another.

(a) What is the number of possible outcomes? (You might want to list the outcomes so that it's easier to count.)

(b) What is the probability of first rolling a 1 and then rolling a 2?

- (c) What is the probability of first rolling a 2 and then rolling a 1?
- (d) What is the probability that both rolls give the same number?
- (e) What is the probability that the number on the second roll is strictly larger than the number on the first roll?
- (f) What is the probability that the first roll is an odd number?

(g) What is the probability that both numbers are odd?

8. (Monty Hall Problem) Suppose you are on a game show, and you're given the choice of three doors: Behind one door is a car and behind the other doors are goats. You pick a door, say door Number 1, and the host, who knows what's behind the doors, opens another door, say door Number 3, which has a goat. He then says to you, "Do you want to pick door Number 2 instead?" Is it to your advantage to switch your choice?