A Game With Dice

January 11, 2015

Suppose we roll two dice at the same time and find the sum of the two numbers we rolled. What answer will we get most often?

Imagine this as a game: your friend rolls two dice and finds the sum of the numbers. You need to predict what sum your friend will get.

If you predict correctly, you win the game! What number should you choose so that you win as often as possible?

1. Construct the addition table for numbers from 1 to 6.

<table>
<thead>
<tr>
<th>+</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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What is the smallest number you can get? What is the largest number
2. Nikki thinks that all the sums are equally likely. Is she correct? Why or why not?

3. Fill in the table below showing how you can get each of the sums (between 2 and 12).

   For example, to get a 3 you need to roll 1 and 2 or 2 and 1 (as shown below).

<table>
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<tr>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1+2</td>
<td>1+3</td>
<td>1+4</td>
<td>1+5</td>
<td>1+6</td>
<td>2+6</td>
<td>3+6</td>
<td>4+6</td>
<td>5+6</td>
<td>6+6</td>
</tr>
</tbody>
</table>

4. What sum are we most likely to get as the sum of the two numbers on the dice?

5. What sums are we least likely to get as the sum of the two numbers on the dice?

6. Are we more likely to get 3 as the sum, or to get 4 as the sum?
7. Make a bar graph below for the number of ways to get a given sum for each of the numbers between 2 and 12.

What do you notice about the shape of your bar graph?

8. Based on the graph, what number is the most likely one to come up as the sum?
9. Now do an experiment! Roll a pair of dice and compute the sum. Repeat this 30 times. In the bar chart below, record how many times you get each sum.

(You can do this with a partner: one of you will throw the dice, the other records the sum on the bar graph. Switch with your partner halfway through.)

10. Based on our investigation, what number should you choose to win the game with dice as often as possible?
Bon us problems! (Problems 1 - 7 below are based on problems from the 2006 Math Kangaroo for 1st and 2nd grade students.)

1. Place the signs +, -, and = between the digits

\[ 4 \quad 8 \quad 3 \quad 3 \quad 6 \quad 9 \]

in such a way that an equation is built. You can change the order of the signs, but each sign is used only once.

2. As Hans wrote the problem on the board, he forgot one digit:

\[ 23 + 31 + 2 + 12 = 94. \]

What digit did Hans forget?

3. The first day of a certain month that has 30 days is a Monday. How many Mondays does this month have?
4. Matthias received a book as a gift. He colored in it on page 99 through 110. How many pages did Matthias color?

5. Helen said to Mary, “If you give me 4 apples, I will have exactly as many apples as you.” How many more apples does Mary have than Helen?

6. Last year, the sum of ages of Iris, Beatrice, and Lydia was 18 years. How old are they together this year?

7. In the class there are 20 students, 8 of whom are boys. At how many desks do the girls sit, if two of them sit at each desk?
8. Bacteria are growing in a laboratory dish. After each hour, every bacteria divides into two, to make two new ones. After 10 hours, the dish is full.

(a) When was the dish half full?

(b) When was the dish one quarter full?

9. Ten thousand pine trees grow in a forest. Scientists have discovered that in this forest, no pine tree has more than 1000 pine needles on it. Do you think there are two pine trees in the forest that have the same number of pine needles? How can you be sure?